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# Asymptotic Behavior Of The Approximation Numbers Of The Hardy-Type Operator From $L^p$ Into $L^q$

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**Abstract:** We consider the Hardy-type operator

$$(Tf)(x) := v(x)\int_a^x u(t)f(t)dt, \qquad x > a,$$

and establish properties of $T$ as a map from $L^p(a,b)$ into $L^q(a,b)$ for $1 < p \le q \le 2$, $2 \le p \le q < \infty$ and $1 < p \le 2 \le q < \infty$. The main result is that, with appropriate assumptions on $u$ and $v$, the approximation numbers $a_n(T)$ of $T$ satisfy the inequality

$$c_1 \int_a^b |uv|^r dt \le \liminf_{n\to\infty} n a_n^r(T) \le \limsup_{n\to\infty} n a_n^r(T) \le c_2 \int_a^b |uv|^r dt$$

when $1 < p \le q \le 2$ or $2 \le p \le q < \infty$, and in the case $1 < p \le 2 \le q < \infty$ we have

$$\limsup_{n\to\infty} n a_n^r(T) \le c_3 \int_0^d |u(t)v(t)|^r dt$$

and

$$c_4 \int_0^d |u(t)v(t)|^r dt \le \liminf_{n\to\infty} n^{(1/2-1/q)r+1} a_n^r(T),$$

where $r = \frac{p'q}{p'+q}$ and constants $c_1, c_2, c_3, c_4$. Upper and lower estimates for the $l^s$ and $l^{s,k}$ norms of $\{a_n(T)\}$ are also given.