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Approximation of $\pi(x)$ by $\Psi(x)$

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Abstract:

In this paper we find some lower and upper bounds of the form $\frac{n}{H_n-c}$ for the function $\pi(n)$, in which $H_n=\sum_{k=1}^{n}\frac{1}{k}$. Then, we consider $H(x)=\Psi(x+1)+\gamma$ as generalization of H_n, such that $\Psi(x)=\frac{d}{dx}\log\Gamma(x)$ and γ is Euler constant; this extension has been introduced for the first time by J. Sándor and it helps us to find some lower and upper bounds of the form $\frac{x}{\Psi(x)-c}$ for the function $\pi(x)$ and using these bounds, we show that $\Psi(p_n)\sim\log n$, when $n\longrightarrow\infty$ is equivalent with the Prime Number Theorem.