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An Inequality Between Compositions of Weighted Arithmetic and Geometric Means

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Abstract:

Let $\mathbb{P}$ denote the collection of positive sequences defined on $\mathbb{N}$. Fix $w \in \mathbb{P}$. Let s, t, respectively, be the sequences of partial sums of the infinite series $\sum w_k$ and $\sum s_k$, respectively. Given $x \in \mathbb{P}$, define the sequences $A(x)$ and $G(x)$ of weighted arithmetic and geometric means of x by

$$A_n(x) = \sum_{k=1}^{n} \frac{w_k}{s_n} x_k, \; G_n(x) = \prod_{k=1}^{n} x_k^{w_k/s_n}, \; n = 1, 2, \ldots$$

Under the assumption that $\log t$ is concave, it is proved that $A(G(x)) \leq G(A(x))$ for all $x \in \mathbb{P}$, with equality if and only if x is a constant sequence.