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A Weighted Analytic Center for Linear Matrix Inequalities

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Abstract:

Let $\mathcal{R}$ be the convex subset of $\mathbb{R}^n$ defined by q simultaneous linear matrix inequalities (LMI) $A_0^{(j)} + \sum_{i=1}^n x_i A_i^{(j)} \succ 0, \quad j = 1, 2, \ldots, q$. Given a strictly positive vector $\omega = (\omega_1, \omega_2, \cdots, \omega_q)$, the **weighted analytic center** $x_{ac}(\omega)$ is the minimizer**argmin** $(\phi_\omega(x))$ of the strictly convex function $\phi_\omega(x) = \sum_{j=1}^q \omega_j \log\det[A^{(j)}(x)]^{-1}$ over $\mathcal{R}$. We give a necessary and sufficient condition for a point of $\mathcal{R}$ to be a weighted analytic center. We study the **argmin** function in this instance and show that it is a continuously differentiable open function.

In the special case of linear constraints, all interior points are weighted analytic centers. We show that the region $\mathcal{W} = \{x_{ac}(\omega) \mid \omega > 0\} \subseteq \mathcal{R}$ of weighted analytic centers for LMI's is not convex and does not generally equal $\mathcal{R}$. These results imply that the techniques in linear programming of following paths of analytic centers may require special consideration when extended to semidefinite programming. We show that the region $\mathcal{W}$ and its boundary are described by real algebraic varieties, and provide slices of a non-trivial real algebraic variety to show that $\mathcal{W}$ isn't convex. Stiemke's Theorem of the alternative provides a practical test of whether a point is in $\mathcal{W}$. Weighted analytic centers are used to improve the location of standing points for the Stand and Hit method of identifying necessary LMI constraints in semidefinite programming.

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