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A New Obstruction to Minimal Isometric Immersions into a Real Space Form

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Abstract: In the theory of minimal submanifolds, the following problem is fundamental: *when does a given Riemannian manifold admit (or does not admit) a minimal isometric immersion into an Euclidean space of arbitrary dimension?* S.S. Chern, in his monograph [6] *Minimal submanifolds in a Riemannian manifold*, remarked that the result of Takahashi (*the Ricci tensor of a minimal submanifold into a Euclidean space is negative semidefinite)* was the only known Riemannian obstruction to minimal isometric immersions in Euclidean spaces. A second obstruction was obtained by B.Y. Chen as an immediate application of his fundamental inequality [1]: *the scalar curvature and the sectional curvature of a minimal submanifold into a Euclidean space satisfies the inequality* $\tau \leq k$. We find a new relation between the Chen invariant, the dimension of the submanifold, the length of the mean curvature vector field and a deviation parameter. This result implies a new obstruction: *the sectional curvature of a minimal submanifold into a Euclidean space also satisfies the inequality* $k \leq -\tau$.