## Mathematics > Combinatorics

## Origami rings

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Motivated by a question in origami, we consider sets of points in the complex plane constructed in the following way. Let \$L_lalpha(p)\$ be the line in the complex plane through \$p\$ with angle \$lalpha\$ (with respect to the real axis). Given a fixed collection $\$ \mathrm{U} \$$ of angles, let $\$ \backslash R U \$$ be the points that can be obtained by starting with $\$ 0 \$$ and $\$ 1 \$$, and then recursively adding intersection points of the form \$L_lalpha(p) Icap L_Ibeta(q)\$, where $\$ p, q \$$ have been constructed already, and \$lalpha, \beta\$ are distinct angles in \$U\$.
Our main result is that if $\$ U \$$ is a group with at least three elements, then $\$ \backslash R U \$$ is a subring of the complex plane, i.e., it is closed under complex addition and multiplication. This enables us to answer a specific question about origami folds: if $\$ n$ lge $3 \$$ and the allowable angles are the $\$ n \$$ equally spaced angles $\$ k / p i / n \$$, $\$ 0 \backslash l e k<n \$$, then $\$ \backslash R U \$$ is the ring $\$ \backslash Z[$ lzeta_n] $\$$ if $\$ n \$$ is prime, and the ring $\$ \backslash Z$ $\left[1 / n, \backslash z e t a \_\{n\}\right] \$$ if $\$ n \$$ is not prime, where $\$ \backslash z e t a \_n:=\backslash \exp (2 \backslash p i \mathrm{i} / n) \$$ is a primitive $\$ \mathrm{n} \$$-th root of unity.

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