Origami rings

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(Submitted on 11 Nov 2010)

Motivated by a question in origami, we consider sets of points in the complex plane constructed in the following way. Let $L_\alpha(p)$ be the line in the complex plane through p with angle α (with respect to the real axis). Given a fixed collection U of angles, let $\RU$ be the points that can be obtained by starting with 0 and 1, and then recursively adding intersection points of the form $L_\alpha(p) \cap L_\beta(q)$, where p, q have been constructed already, and α, β are distinct angles in U.
Our main result is that if U is a group with at least three elements, then $\RU$ is a subring of the complex plane, i.e., it is closed under complex addition and multiplication. This enables us to answer a specific question about origami folds: if $n \ge 3$ and the allowable angles are the n equally spaced angles $k\pi/n$, $0 \le k < n$, then $\RU$ is the ring $\Z[\zeta_n]$ if n is prime, and the ring $\Z [1/n,\zeta_{n}]$ if n is not prime, where $\zeta_n := \exp(2\pi i/n)$ is a primitive n-th root of unity.

Comments:	12 pages, 4 figures
Subjects:	**Combinatorics (math.CO)**; Number Theory (math.NT)
MSC classes:	`11R04, 11R18, 00A08`
Cite as:	**arXiv:1011.2769v1 [math.CO]**

Submission history

From: Steve Butler [view email]
[v1] Thu, 11 Nov 2010 21:06:35 GMT (93kb)