Mathematics > Differential Geometry

Degeneration of Kaehler structures and half-form quantization of toric varieties

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(Submitted on 15 Nov 2010)

We study the half-form Kaehler quantization of a smooth symplectic toric manifold (X,ω), such that $[\omega/2\pi]-c_{1}(X)/2 \in H^{2}(X,{\mathbb{Z}})$ and is nonnegative. We define the half-form corrected quantization of (X,ω) to be given by holomorphic sections of a certain hermitian line bundle $L\rightarrow X$ with Chern class $[\omega/ 2\pi]-c_{1}(X)/2$. These sections then correspond to integral points of a "corrected" polytope P_{L} with integral vertices. For a suitably translated moment polytope P_{X} for (X,ω), we have that $P_{L}\subset P_{X}$ is obtained from P_{X} by a one-half inward-pointing normal shift along the boundary.

We use our results on the Kaehler quantization to motivate a definition of half-form corrected quantization in the singular real toric polarization. Using families of complex structures studied in [Baier-Florentino-Mourao-Nunes:arXiv/0806.0606], which include the degeneration of Kaehler polarizations to the vertical polarization, we show that, under this degeneration, the half-form corrected L^{2}-normalized monomial holomorphic sections converge to Dirac-delta-distributional sections supported on the fibers over the integral points of P_{L}, which correspond to corrected Bohr-Sommerfeld fibers. This result and the limit of the corrected connection, with curvature singularities along the boundary of P_X, justifies the direct definition we give for the corrected quantization in the singular real toric polarization. We show that the space of quantum states for this definition coincides with the space obtained via degeneration of the K\"ahler quantization.

We also show that the BKS pairing between Kaehler polarizations is not unitary in general. On the other hand, the unitary connection induced by this pairing is flat.

Comments:	32 pages
Subjects:	**Differential Geometry (math.DG)**
MSC classes:	53D50, 14M25, 32G05
Cite as:	**arXiv:1011.3363v1 [math.DG]**

Submission history

From: William D. Kirwin [view email]
[v1] Mon, 15 Nov 2010 12:42:32 GMT (33kb)

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