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λ-central BMO Estimates for Multilinear Commutators Generated by Calderón-Zygmund Operator

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Abstract: In this paper presents the procedure in which the author obtains the boundedness on central morrey spaces $\dot{B}^{q,\lambda}(R^n)$ for the multilinear commutators $T_{\vec{b}}$ generated by Calderón-Zygmund operator and vector symbol $\vec{b}=(b_1,\cdots,b_m)$, where b_i is a λ-central BMO function.

Key words: multilinear commutators, Calderón-Zygmund operator, central Morrey spaces, λ-central BMO

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1 Introduction and Statements of Results

As one of the most important operators in harmonic analysis and its applications in some related topics, the Calderón-Zygmund operator are of interest in analysis.

Let $K(x,y)$ be a function satisfing the following size condition:

$$|K(x,y)|\leqslant C|x-y|^{-n},\ x\neq y, \tag{1}$$

and if $|x-y|\geqslant 2|x-x'|$, $|K(x,y)-K(x',y)|+|K(y,x)-K(y,x')|\leqslant C|x-x'|^{\delta}/|x-y|^{n+\delta}$, where $0<\delta\leqslant 1$ and $C>0$.

The Calderón-Zygmund operator associated to the above kernel K is formally defined by $Tf(x)=\int_{R^n}K(x,y)f(y)\,\mathrm{d}y$, where x is not in the support of f. A celebrated result is that T is a bounded oper-ator on $L^p(R^n)$ space when $1<p<\infty$.

On the other hand, in 1976, Coifman, et al[1] introduced the commutator T_b generated by a function b and the Calderón-Zygmund operator T, which is defined by

$$T_b(f)(x)=\int_{R^n}K(x,y)[b(x)-b(y)]f(y)\,\mathrm{d}y.$$

The main result from [1] states that T_b is a bounded operator on $L^p(R^n)$, $1<p<\infty$, when the symbol b is a BMO function. It is natural to ask whether the above conclusion also be right when b is in more generally space than BMO. In 2002, Alvarez, et al[2] introduced a kind of more generally space than BMO, i.e., λ-central bounded mean oscillation spaces.

Definition 1[2] Given $\lambda<1/n$, $1<q<\infty$, the λ-

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central bounded mean oscillation space $CBMO^{q,\lambda}(R^n)$ is defined as

$$CBMO^{q,\lambda}(R^n)=\{f\in L^q_{loc}(R^n):\|f\|_{CBMO^{q,\lambda}}<\infty\},$$

where the norm $\|f\|_{CBMO^{q,\lambda}}$ is given by

$$\|f\|_{CBMO^{q,\lambda}}=\sup_{R>0}\Big(\int_{B(0,R)}|f(x)-f_{B(0,R)}|^q\,\mathrm{d}x/|B(0,R)|^{1+q\lambda}\Big)^{1/q}, \qquad (2)$$

here and in what follows, $B(x,R)$ denotes the ball in R^n with center x and radius R, $|B(x,R)|$ denotes its volume and $f_{B(0,R)}=\int_{B(0,R)}f(x)\,\mathrm{d}x/|B(0,R)|$.

Obviously, when $\lambda=0$, $CBMO^{q,\lambda}(R^n)=CBMO^{q}(R^n)$ defined in [3]. In addition, we know $BMO\subset\cap_{q>1}CBMO^{q}$ (also see [4]). Hence, if we only assume $b\in CBMO^{q}$, or more generally $b\in CBMO^{q,\lambda}$, T_b may not be a bounded operator on $L^p(R^n)$. However, it has some boundedness on other spaces. As a matter of fact, Alvarez et al[2] and Komori[5] have obtained the λ-central BMO estimates for the commutators of a class of singular integral operators on central Morrey spaces.

Let us first recall the definition of central Morrey space.

Definition 2[2] Let $\lambda\in R$, $1<q<\infty$, the central Morrey space $\dot{B}^{q,\lambda}(R^n)$ is defined as

$$\dot{B}^{q,\lambda}(R^n)=\{f\in L^q_{loc}(R^n):\|f\|_{\dot{B}^{q,\lambda}}<\infty\},$$

where the norm $\|f\|_{\dot{B}^{q,\lambda}}$ is given by

$$\|f\|_{\dot{B}^{q,\lambda}}=\sup_{R>0}\Big(\int_{B(0,R)}|f(x)|^q\,\mathrm{d}x/|B(0,R)|^{1+q\lambda}\Big)^{1/q}. \qquad (3)$$

Here, we state the corresponding result in [2,5] of T_b as follows (also see [4]).

Proposition 1 Let $1<q<p<\infty$, $p'<l<\infty$, $1/q=1/p+1/l$, $0<\mu<1/n$, $\lambda=\lambda_1+\mu<0$. If b is in $CBMO^{l,\mu}$ and T is the Calderón-Zygmund operator, then there exists a constant $C>0$ which is independent of f such that

$$\|T_b(f)\|_{\dot{B}^{q,\lambda}}\leqslant C\|b\|_{CBMO^{l,\mu}}\|f\|_{\dot{B}^{p,\lambda_1}}.$$

For the sake of generalizing the above result to a more general case. Fu, et al[4], in 2008, discussed the operator $T_{b,\Omega}$ with rough kernel Ω on central Morrey space $\dot{B}^{q,\lambda}(R^n)$. In this paper, we will focus on discussing λ-central BMO estimates on central Morrey spaces for a wider class of commutators, which is introduced by Pérez et al[6] in 2002, i.e., the multilinear commutators $T_{\vec{b}}$ with vector symbol $\vec{b}=(b_1,\cdots,b_m)$ defined by,

$$T_{\vec{b}}(f)(x)=\int_{R^n}K(x,y)f(y)\prod_{i=1}^{m}(b_i(x)-b_i(y))\,\mathrm{d}y, \qquad (4)$$

where T is Calderón-Zygmund operator associated K satisfying the condition (1) and some smoothness assumption. We obtain the following theorem.

Theorem 1 Let T is a the Calderón-Zygmund operator associated K satisfying the condition (1), $T_{\vec{b}}$ is the multilinear commutators defined by (4). For any $i\in N$, suppose that $0<\mu_i<1/n$, $1<q<p_1<\infty$, $1<p'_1<l_i<\infty$, $1/q=\sum_{i=1}^{m}1/l_i+1/p_1$, $\lambda=\lambda_1+\sum_{i=1}^{m}\mu_i<0$. If $b_i\in CBMO^{l_i,\mu_i}(R^n)$, then $T_{\vec{b}}$ is bounded from $\dot{B}^{p_1,\lambda_1}(R^n)$ to $\dot{B}^{q,\lambda}(R^n)$ and

$$\|T_{\vec{b}}(f)\|_{\dot{B}^{q,\lambda}}\leqslant C\prod_{i=1}^{m}\|b_i\|_{CBMO^{l_i,\mu_i}}\|f\|_{\dot{B}^{p_1,\lambda_1}}, \qquad (5)$$

where C is a constant which is independent of f.

Throughout this paper, the letter C always remains to denote a positive constant that may varies at each occurrence but is independent of the essential variable.

2 Proof of Theorem 1

Our proof of this theorem depends on a little technical. The spirit is the estimates for the operator $T_{\vec{b}}$ are treated on the local parts and the nonlocal parts respectively. We use $L^p(R^n)$ boundedness given already in last section for Calderón-Zygmund operator T. Without loss of generality, we assume that $m=2$. For fixed $R>0$, denote $B(0,R)$ by B and $B(0,kR)$ by kB for $k\in N$, assume that $f\in \dot{B}^{p_1,\lambda_1}(R^n)$, we write

$$f(x)=f(x)\chi_{2B}+f(x)(1-\chi_{2B}):=f_1(x)+f_2(x),$$

and

$$T_{\vec{b}}(f)(x)=[b_1(x)-b_{1\,B}][b_2(x)-b_{2\,B}]Tf(x)-[b_1(x)-b_{1\,B}]T[(b_2(\cdot)-b_{2\,B})f](x)-[b_2(x)-b_{2\,B}]T[(b_1(\cdot)-b_{1\,B})f](x)+T[(b_1(\cdot)-b_{1\,B})(b_2(\cdot)-b_{2\,B})f](x),$$

where χ_{2B} is the characteristic function of ball $B(0,2R)$ and $b_{i\,B}$ denotes the average of the function b_i over the ball B. Thus, the following integral can be written by

$$\begin{aligned}(\int_B|T_{\vec{b}}(f)(x)|^q\,\mathrm{d}x)^{1/q}\leqslant\ &(\int_B|[b_1(x)-b_{1\,B}][b_2(x)-b_{2\,B}]Tf(x)|^q\,\mathrm{d}x)^{1/q}+\\&(\int_B|[b_1(x)-b_{1\,B}]T[(b_2(\cdot)-b_{2\,B})f](x)|^q\,\mathrm{d}x)^{1/q}+\\&(\int_B|[b_2(x)-b_{2B}]T[(b_1(\cdot)-b_{1\,B})f](x)|^q\,\mathrm{d}x)^{1/q}+\\&(\int_B|T[b(\cdot)-b_{1B}][(b_2(\cdot)-b_{2B})f](x)|^q\,\mathrm{d}x)^{1/q}:=\\&J_1+J_2+J_3+J_4.\end{aligned}\qquad(6)$$

(i) We consider the estimate of J_1 as follows:

$$\begin{aligned}J_1\leqslant&(\int_B|[b_1(x)-b_{1\,B}][(b_2(x)-b_{2\,B}]Tf_1(x)|^q\,\mathrm{d}x)^{1/q}+\\&(\int_B|[b_1(x)-b_{1\,B}][(b_2(x)-b_{2\,B}]Tf_2(x)|^q\cdot\mathrm{d}x)^{1/q}:=J_{11}+J_{12}.\end{aligned}$$

Firstly, applying the Hölder inequality with exponents q and then by the boundedness of T on $L^q(R^n)$, we have

$$\begin{aligned}J_{11}\leqslant&C\prod_{i=1}^{2}(\int_B\left|b_i(x)-b_{i\,B}\right|^{l_i}\mathrm{d}x)^{1/l_i}(\int_B\left|Tf_1(x)\right|^{p_1}\mathrm{d}x)^{1/p_1}\leqslant\\&C|B|^{1/q+\lambda}\|b_1\|_{CBMO^{l_1,\mu_1}}\|b_2\|_{CBMO^{l_2,\mu_2}}\|f\|_{\dot{B}^{p_1,\lambda_1}}.\end{aligned}$$

To estimate the term of J_{12}, we first note that $|x-y|^n\sim|2^{k-1}B|$ for $x\in B$ and $y_i\in(2^kB)^c$. By the condition (1), $\lambda=\lambda_1+\sum_{i=1}^{m}\mu_i<0$ and the Hölder inequality, we get

$$\begin{aligned}|Tf_2(x)|=&\sum_{k=1}^{+\infty}\int_{2^{k+1}B\backslash 2^kB}|f(y)|/|x-y|^n\,\mathrm{d}y\leqslant\\&C\sum_{k=1}^{+\infty}|2^{k-1}B|^{-1}\int_{2^{k+1}B}|f(y)|\,\mathrm{d}y\leqslant\\&C|B|^{\lambda_1}\|f\|_{\dot{B}^{p_1,\lambda_1}}(\sum_{k=1}^{+\infty}2^{n\lambda_1})\leqslant\\&C|B|^{\lambda_1}\|f\|_{\dot{B}^{p_1,\lambda_1}}.\end{aligned}$$

So, the Hölder inequality applied to the J_{12} yields that

$$J_{12}\leqslant C|B|^{1/q+\lambda}\|b_1\|_{CBMO^{l_1,\mu_1}}\|b_2\|_{CBMO^{l_2,\mu_2}}\|f\|_{\dot{B}^{p_1,\lambda_1}},$$

Therefore,

$$J_1\leqslant C|B|^{1/q+\lambda}\|b_1\|_{CBMO^{l_1,\mu_1}}\|b_2\|_{CBMO^{l_2,\mu_2}}\|f\|_{\dot{B}^{p_1,\lambda_1}}.\qquad(7)$$

(ii) Now consider J_2, denote $1/q_1=1/l_2+1/p_1$, then $1/q=1/l_1+1/q_1$, by the Hölder inequality, we have

$$\begin{aligned}J_2\leqslant&C|B|^{1/l_1+\mu_1}\|b_1\|_{CBMO^{l_1,\mu_1}}\times\\&(\int_B|T[(b_2(\cdot)-b_{2B})f](x)|^{q_1}\,\mathrm{d}x)^{1/q_1}\leqslant\\&C|B|^{1/l_1+\mu_1}\|b_1\|_{CBMO^{l_1,\mu_1}}(J_{21}+J_{22}),\end{aligned}\qquad(8)$$

where $J_{21}:=(\int_B|T[(b_2(\cdot)-b_{2\,B})f_1](x)|^{q_1}\,\mathrm{d}x)^{1/q_1}$, and $J_{22}:=(\int_B|T[(b_2(\cdot)-b_{2\,B})f_2](x)|^{q_1}\mathrm{d}x)^{1/q_1}$.

For J_{21}, since $1<p'_1<l_2<\infty$, we know $q_1>1$. First of all, using the boundedness of T on the space $L^{q_1}(R^n)$, and then from the Hölder inequality with exponents q_1, we have

$$\begin{aligned}J_{21}\leqslant&C\|(b_2(.)-b_{2B})f_1\|_{L^{q_1}(R^n)}\leqslant\\&C(\int_{2B}|b_2(x)-b_{2\,B}|^{l_2}\,\mathrm{d}x)^{1/l_2}(\int_{2B}|f(x)|^{p_1}\cdot\\&\mathrm{d}x)^{1/p_1}\leqslant C|B|^{1/p_1+\lambda_1}\|f\|_{\dot{B}^{p_1,\lambda_1}}[(\int_{2B}|b_2(x)-\\&b_{2\,2B}|^{l_2}\,\mathrm{d}x)^{1/l_2}+\|2B\|^{1/l_2}|b_{2\,2B}-b_{2\,B}|]\leqslant\\&C|B|^{1/p_1+\lambda_1}|B|^{1/l_2+\mu_2}\|f\|_{\dot{B}^{p_1,\lambda_1}}\|b_2\|_{CBMO^{l_2,\mu_2}}.\end{aligned}$$

In fact, to get the inequality above, the following fact is applied

$$|b_{2\,2B}-b_{2\,B}|\leqslant 2^n/|2B|\int_{2B}|b_2(x)-b_{2\,2B}|\,\mathrm{d}x\leqslant$$
$$C/|2B|^{1/l_2}\left(\int_{2B}|b_2(x)-b_{2\,2B}|^{l_2}\,\mathrm{d}x\right)^{1/l_2}. \quad (9)$$

Now we treat term J_{22}. Similar with the estimate of J_{12}, the fact $|x-y|^n\sim|2^{k-1}B|$ for $x\in B$ and $y_i\in(2^kB)^c$ is also used. Using the condition (1), the Minkowski inequality, the Hölder inequality with exponents q_1, and $\lambda=\lambda_1+\sum_{i=1}^m\mu_i<0$, we get

$$J_{22}\leqslant C\{\int_B(\int_{(2B)^c}|(b_2(y)-b_{2\,B})f(y)|/|x-y|^n\,\mathrm{d}y)^{q_1}\mathrm{d}x\}^{1/q_1}\leqslant C\int_{(2B)^c}(\int_B\|(b_2(y)-b_{2\,B})f(y)|/|x-y|^n|^{q_1}\mathrm{d}x)^{1/q_1}\mathrm{d}y\leqslant$$
$$C\sum_{k=1}^{\infty}\int_{2^{k+1}B\setminus 2^kB}(\int_B\|(b_2(y)-b_{2\,B})\|f(y)|/|x-y|^n|^{q_1}\mathrm{d}x)^{1/q_1}\mathrm{d}y\leqslant$$
$$C\sum_{k=1}^{\infty}2^{-kn/q_1}(\int_{2^{k+1}B}|(b_2(y)-b_{2\,B})|^{l_2}\,\mathrm{d}y)^{1/l_2}\cdot(\int_{2^{k+1}B}|f(y)|^{p_1}\,\mathrm{d}y)^{1/p_1}\leqslant$$
$$C\sum_{k=1}^{\infty}[(\int_{2^{k+1}B}|(b_2(y)-b_{2\,2^{k+1}B})|^{l_2}\,\mathrm{d}y)^{1/l_2}+|2^{k+1}B|^{1/l_2}|b_{2\,2^{k+1}B}-b_{2\,B}|]2^{-kn/q_1}|2^{k+1}B|^{1/p_1+\lambda_1}\cdot$$
$$\|f\|_{\dot{B}^{p_1,\lambda_1}}\leqslant C\sum_{k=1}^{\infty}2^{-kn/q_1}|2^{k+1}B|^{1/l_2+\mu_2 1/p_1+\lambda_1}\cdot$$
$$\|b_2\|_{CBMO^{l_2,\mu_2}}\|f\|_{\dot{B}^{p_1,\lambda_1}}\leqslant C|B|^{1/q_1+\mu_2+\lambda_1}\cdot$$
$$\|b_2\|_{CBMO^{l_2,\mu_2}}\|f\|_{\dot{B}^{p_1,\lambda_1}}(\sum_{k=1}^{\infty}2^{kn(\lambda_1+\mu_2)})\leqslant$$
$$C|B|^{1/q_1+\mu_2+\lambda_1}\|b_2\|_{CBMO^{l_2,\mu_2}}\|f\|_{\dot{B}^{p_1,\lambda_1}},$$

as desired.

Notice that, the estimation above depends on the following fact, which is coincided with the inequality (9) :

$$|b_{2\,2^{k+1}B}-b_{2\,B}|\leqslant C\sum_{j=0}^{k}|2^{j+1}B|^{-1/l_2}$$
$$(\int_{2^{j+1}B}|b_2(x)-b_{2\,2^{j+1}B}|^{l_2}\,\mathrm{d}x)^{1/l_2}\leqslant$$
$$C\|b_2\|_{CBMO^{l_2,\mu_2}}|2^{k+1}B|^{\mu_2}. \quad (10)$$

Combining the estimates for J_{21}, J_{22} and (8), we obtain

$$J_2\leqslant C|B|^{1/q+\lambda}\|b_1\|_{CBMO^{l_1,\mu_1}}\|b_2\|_{CBMO^{l_2,\mu_2}}\|f\|_{\dot{B}^{p_1,\lambda_1}}, \quad (11)$$

as desired.

Likewise, the estimates of J_3 is symmetric to J_2, therefore, the estimate of J_3 following that

$$J_3\leqslant C|B|^{1/q+\lambda}\|b_1\|_{CBMO^{l_1,\mu_1}}\|b_2\|_{CBMO^{l_2,\mu_2}}\|f\|_{\dot{B}^{p_1,\lambda_1}}. \quad (12)$$

(iii) Finally, we have to consider J_4. We split this term similarly:

$$J_4\leqslant(\int_B|T[(b_1(\cdot)-b_{1\,B})(b_2(\cdot)-b_{2\,B})f_1](x)|^q\,\mathrm{d}x)^{1/q}+(\int_B|T[(b_1(\cdot)-b_{1\,B})(b_2(\cdot)-b_{2\,B})f_2](x)|^q\cdot\mathrm{d}x)^{1/q}:=J_{41}+J_{42}. \quad (13)$$

Consider J_{41} first. The estimation of J_{41} is the same as before, since T is a bounded operator on $L^q(R^n)$, moreover the Hölder inequality with exponents q imply

$$J_{41}\leqslant C\|(b_1(\cdot)-b_{1\,B})(b_2(\cdot)-b_{2\,B})f_1\|_{L^q(R^n)}\leqslant$$
$$C\prod_{i=1}^{2}(\int_{2B}|b_i(x)-b_{i\,B}|^{l_i}\,\mathrm{d}x)^{1//l_i}\cdot$$
$$(\int_{2B}|f(x)|^{p_1}\mathrm{d}x)^{1//p_1}\leqslant C|B|^{1/q+\lambda}\times$$
$$\|b_1\|_{CBMO^{l_1,\mu_1}}\|b_2\|_{CBMO^{l_2,\mu_2}}\|f\|_{\dot{B}^{p_1,\lambda_1}}.$$

Notice that here we use the same method during the estimate of J_{21} and (9) for every function b_i.

Next we consider J_{42}. The spirit is similar to that of J_{22}, by the fact $|x-y|^n\sim|2^{k-1}B|$ for $x\in B$ and $y\in(2^kB)^c$, the Minkowski inequality, the Hölder inequality with exponents q, the inequality (10) and the condition $\lambda=\lambda_1+\sum_{i=1}^m\mu_i<0$, we get

$$J_{42}\leqslant C\{\int_B(\int_{(2B)^c}|(b_1(y)-b_{1\,B})(b_2(y)-b_{2\,B})f(y)|/|x-y|^n\,\mathrm{d}y)^q\,\mathrm{d}x\}^{1/q}\leqslant C\int_{(2B)^c}(\int_B\|(b_1(y)-b_{1\,B})(b_2(y)-b_{2\,B})f(y)|/|x-y|^n|^q\,\mathrm{d}x)^{1/q}\mathrm{d}y\leqslant$$
$$C\sum_{k=1}^{\infty}\int_{2^{k+1}B\setminus 2^kB}(\int_B\|(b_1(y)-b_{1\,B})(b_2(y)-b_{2\,B})\cdot$$

$$f(y)|/|x-y|^{n}|^{q}\,\mathrm{d}x)^{1/q}\,\mathrm{d}y \leqslant C\sum_{k=1}^{\infty}2^{-kn/q}\cdot$$

$$\prod_{i=1}^{2}\Big(\int_{2^{k+1}B}|(b_i(y)-b_{i\,B})|^{l_i}\mathrm{d}y\Big)^{1/l_i}\Big(\int_{2^{k+1}B}|f(y)|^{p_1}\cdot$$

$$\mathrm{d}y)^{1/p_1}C\sum_{k=1}^{\infty}(2^{-kn/q_1}\,|2^{k+1}B|^{1/p_1+\lambda_1}\|f\|_{\dot{B}^{p_1,\lambda_1}})\cdot$$

$$\Big\{\prod_{i=1}^{2}\Big(\int_{2^{k+1}B}|(b_i(y)-b_{i\,2^{k+1}B})|^{l_i}\,\mathrm{d}y\Big)^{1/l_i}+$$

$$|2^{k+1}B|^{1/l_i}|b_{i\,2^{k+1}B}-b_{i\,B}|\Big\}\leqslant C|B|^{1/q+\lambda}\cdot$$

$$\|b_1\|_{CBMO^{l_1,\mu_1}}\|b_2\|_{CBMO^{l_2,\mu_2}}\|f\|_{\dot{B}^{p_1,\lambda_1}}\Big(\sum_{k=1}^{\infty}2^{kn\lambda}\Big)\leqslant$$

$$C|B|^{1/q_1+\mu_2+\lambda_1}\|b_2\|_{CBMO^{l_2,\mu_2}}\|f\|_{\dot{B}^{p_1,\lambda_1}}.$$

Thus, the estimates above and (13) imply

$$J_4\leqslant C|B|^{1/q+\lambda}\|b_1\|_{CBMO^{l_1,\mu_1}}\|b_2\|_{CBMO^{l_2,\mu_2}}\|f\|_{\dot{B}^{p_1,\lambda_1}}. \tag{14}$$

Finally, a combination of (7), (11), (12) and (14) finish the proof of Theorem 1. This completes the proof of the Theorem 1.

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Calderón-Zygmund 算子多线性交换子的 λ-中心 BMO 估计

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摘要： 建立了由 Calderón-Zygmund 算子和 λ-中心 BMO 函数族生成的多线性交换子 $T_{\vec{b}}$ 在中心 Morrey 空间 $\dot{B}^{q,\lambda}(R^n)$ 上的有界估计，其中 $\vec{b}=(b_1,\cdots,b_m)$，b_i 是 λ-中心 BMO 函数.

关键词： 多线性交换子; Calderón-Zygmund 算子; 中心 Morrey 空间; λ-中心 BMO

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