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Notes on Level Function of Fuzzy Sets

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Abstract: This short note presents a counterexample showing that a result from level function of fuzzy sets in Wu Hsienchung is incorrect, and a correct proof on the convexity of the given fuzzy set is given.

Key words: fuzzy set; upper semicontinuity; convex; resolution identity

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The object of this note is to construct a counter-example to Proposition 2.11 in Wu's paper [1], which also appeared in the papers [2], [3] and [4]. Since proposition 2.11 is mainly used to prove convexity of the resulting fuzzy set, we also give a correct proof on the convexity of the resulting fuzzy set. In the following we use the same notations as in [1].

First we recall that for $f(x)$ a real valued function on a topological space, if the set $\{x : f(x) \geq \alpha\}$ is closed for every α, then $f(x)$ is called up- per semicontinuous. In Wu[1], the following proposi- tion is on level function of fuzzy sets is given:

Proposition 2.11 of [1].

(1) Let $\{A_\alpha = [l_\alpha, u_\alpha] : 0 \leq \alpha \leq 1\}$ be decreesing with respect to α, then $f(\alpha) = \alpha 1_{A_\alpha}(r)$ is upper semicontinuous for any fixed r.

(2) If $\sup\limits_{0\leq\alpha\leq 1} \alpha 1_{A_\alpha}(r) \geq \beta$ then $\exists \alpha_0 \geq \beta$ such that $r \in A_{\alpha_0}$.

Now we show that this proposition is invalid by giving the following counterexample.

Example Let

$$A_\alpha = \begin{cases} [1,3], & 0 < \alpha < \frac{1}{2}, \\ [1,2], & \frac{1}{2} \leq \alpha \leq 1, \end{cases}$$

let $r = 2.5$.

We can see that

$$f(\alpha) = \alpha 1_{A_\alpha}(2.5) = \begin{cases} \alpha, & 0 < \alpha < \frac{1}{2}, \\ 0, & \frac{1}{2} \leq \alpha \leq 1, \end{cases}$$

So

$$\{\alpha : f(\alpha) \geq y\} = \left[y, \frac{1}{2} \right),$$

which is not closed.

We can also see that $\sup\limits_{0\leq\alpha\leq 1} \alpha 1_{A_\alpha}(2.5) = 1/2$. Let $\beta = 1/2$, $\beta \leq 1/2$, for arbitrary $\alpha_0 \geq \beta$, $2.5 \notin A_{\alpha_0}$, thus the above counterexample also shows that (ii) of the proposition 2.11 in [1] is false.

Actually, by the resolution identity [5], the level

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set of $\widetilde{A}$ which is induced by A_α is

$$\widetilde{A}_\alpha = \bigcap_{\lambda<\alpha} A_\lambda \begin{cases} [1,3], & 0<\alpha\le\frac{1}{2}, \\ [1,2], & \frac{1}{2}<\alpha\le 1, \end{cases}$$

Remark In paper [1], proposition 2.11 is only used to prove Proposition 2.12. If $\{A_\alpha=[l_\alpha,u_\alpha]:0\le \alpha\le 1\}$ is decreasing with respect to α, then the fuzzy set $\widetilde{A}$ induced by A_α is a convex fuzzy set. In fact, proposition 2.12 is obvious since $A_\alpha=[l_\alpha, u_\alpha]$ is both closed and convex, and as we know that if A_α is convex set, then $\bigcap_\alpha A_\alpha$ is also convex. Thus the fuzzy set $\widetilde{A}$ induced by A_α is a convex fuzzy set.

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关于模糊集合的层次集函数的注记

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摘要：给出Wu Hsienchung文章中关于模糊集合层次集函数命题的一个反例，同时给出了模糊集合层次集函数有关性质的正确证明.

关键词：模糊集合；上半连续；凸；分解定理

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