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An Algorithm to Recognise Small Seifert Fiber Spaces

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Abstract: The homeomorphism problem is, given two compact n-manifolds, is there an algorithm to decide if the manifolds are homeomorphic or not. The homeomorphism problem has been solved for many important classes of 3-manifolds - especially those with embedded 2-sided incompressible surfaces (cf [12], [15], [16]), which are called Haken manifolds. It is also well-known that the homeomorphism problem is easily solvable for two 3-manifolds which admit geometries in the sense of Thurston [36], [31]. Hence the recognition problem, to decide if a 3-manifold has a geometric structure, is a significant problem. The recognition problem has been solved for all geometric classes, except for the class of small Seifert fibered spaces, which either have finite fundamental group or have fundamental groups which are extensions of Z by a triangle group and have finite abelianisation. Our aim in this paper is to give an algorithm to recognise these last classes of 3-manifolds, i.e to decide if a given 3-manifold is homeomorphic to one in this class. A completely different solution has been announced recently by Tao Li [22]. Also Perelman's announcement of a solution of the geometrisation conjecture would enable a complete solution of the homeomorphism problem; by identifying which geometric structure a given manifold admits. However it is worth noting that practical algorithms for the homeomorphism and recogntion problems, which can be implemented via software, are very useful for experimentation in 3-manifold topology. (See for example [5], [39]).

Key Words: small Seifert fibered space, recognition algorithm, Heegaard splitting, almost normal surface

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