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On the Centroid of the Prime Gamma Rings II

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Abstract: The aim of this paper is to study the properities of the extended centroid of the prime Γ-rings. Main results are the following theorems: (1) Let M be a simple Γ-ring with unity. Suppose that for some $a \neq 0$ in M we have $a\gamma_1 x\gamma_2 a\beta_1 y\beta_2 a = a\beta_1 y\beta_2 a\gamma_1 x\gamma_2 a$ for all x, $y \in$ M and $\gamma_1, \gamma_2, \beta_1, \beta_2 \in \Gamma$. Then M is isomorphic onto the Γ-ring $D_{n,m}$, where $D_{n,m}$ is the additive abelian group of all rectangular matrices of type $n \times m$ over a division ring D and Γ is a nonzero subgroup of the additive abelian group of all rectangular matrices of type $m \times n$ over a division ring D. Furthermore M is the Γ-ring of all $n \times n$ matrices over the field C_Γ. (2) Let M be a prime Γ-ring and C_Γ the extended centroid of M. If a and b are non-zero elements in $S = M\Gamma C_\Gamma$ such that $a\gamma x\beta b = b\beta x\gamma a$ for all $x \in$ M and $\beta, \gamma \in \Gamma$, then a and b are C_Γ-dependent. (3) Let M be prime ΓF-ring, Q quotient Γ-ring of M and C_Γ the extended centroid of M. If q is non-zero element in Q such that $q\gamma_1 x\gamma_2 q\beta_1 y\beta_2 q = q\beta_1 y\beta_2 q\gamma_1 x\gamma_2 q$ for all x, $y \in$ M, $\gamma_1, \gamma_2, \beta_1, \beta_2 \in \Gamma$ then S is a primitive Γ-ring with minimal right (left) ideal such that $e\Gamma$ S, where e is idempotent and $C_\Gamma\Gamma$ e is the commuting ring of S on $e\Gamma$ S.

Key Words: Γ-division ring, Γ-field, extented centroid, central closure.