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Characterization of Some Rings By Functor Z*(.)

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Abstract: Let X = {M : Z*(M) = 0} and X* = {M : Q <= P<=M, P/Q (BAK) X implies P/Q = 0} be classes of R-modules. In this note we study the structure of rings R over which every module M has a decomposition M = M_1 (+) M_2 with M_1 ε X and M_2 ε X*. Let R be a ring with identity. Throughout all modules will be unital right Rmodules and RadM, E(M), Z(M) will denote the radical, injective hull and singular submodule of a module M.J(R) is the Jacobson radical of R. A module N is called a small sub module in a module M if whenever N + L = M for some sub module L of M we have M = L. A module M is said to be small if M is small in E(M). Let M be an R-module. We set Z*(M) = {m ε M : mR is small} and we define inductively Z_n*(M) : Z_1*(M) = Z*(M), Z*(M/Z*{$_n$-1} (M)) = Z*$_n$(M)/Z*{n-1} (M)(n = 2,3,...). It is well-known that Z_2(M) = Z_3(M) = ... for Z(M). But it is not known in Z*$_2$(M) = Z_3M) = . . . In this note we consider the classes X = {M: MR- module and Z*(M) = O},X* = {M : MR- module and whenever Q <= P <= M,P/Q ε X implies P/Q = 0}, following [5]. Since RadM is the sum of small sub modules of M, then RadM <=Z* (M). A class Ω of modules is called s-closed if Ω is closed under sub modules and q-closed if Ω is closed under homomorphic images, and {s, q} -closed if Ω is s-closed and q-closed. It is known that X* is {s,q}-closed. Let Hx(M) denote the sum of X* -sub modules of M. Then Hx(M) ε X*, Hx(M/ Hx (M)) = 0, and Hx is fully invariant [5], and X () X* = 0. It is known that the class X is closed under submodules, direct products, direct sums, essential extensions and module extensions. In [9] it is proved that if R is a quasi-Frobenius ring then every module is a direct sum of an injective module and a small module. In this note we show that every module M over a quasi-Frobenius ring has a decomposition M = M_1 (+) M_2 with M_1 ε X and M_2 ε X*. We also deal with the question: Let R be a ring such that every module M has a decomposition M = M_1 (+) M_2 with M_1 ε X and M_2 ε X*, then R is quasi-Frobenius?

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