Nonlinear Sciences > Pattern Formation and Solitons

Band-edge solitons, Nonlinear Schrodinger / Gross-Pitaevskii Equations and Effective Media

Boaz Ilan, Michael I. Weinstein

(Submitted on 9 Feb 2010 (v1), last revised 18 Feb 2010 (this version, v2))

We consider a class of nonlinear Schrodinger / Gross-Pitaevskii (NLS/GP) equations with periodic potentials, having an even symmetry. We construct "solitons", centered about any point of symmetry of the potential. For focusing (attractive) nonlinearities, these solutions bifurcate from the zero state at the lowest band edge frequency, into the semi-infinite spectral gap. Our results extend to bifurcations into finite spectral gaps, for focusing or defocusing (repulsive) nonlinearities under more restrictive hypotheses. Soliton nonlinear bound states with frequencies near a band edge are well-approximated by a slowly decaying solution of a homogenized NLS/GP equation, with constant homogenized effective mass tensor and effective nonlinear coupling coefficient, modulated by a Bloch state. For the critical NLS equation with a periodic potential, e.g. the cubic two dimensional NLS/GP with a periodic potential, our results imply that the limiting soliton power, as the spectral band edge frequency is approached, is equal to a constant \zeta_* times the minimal mass soliton of the translation invariant critical NLS equation. \zeta_* is expressible in terms of the band edge Bloch eigenfunction and the determinant of the effective mass tensor; and $0<\zeta_*<1$ for any non-constant potential. The results are confirmed by numerical computation of bound states with frequencies near the spectral band edge. Finally, these results have implications for the control of nonlinear waves using periodic structures.

Comments:	57 pages, 6 figures, to be published in SIAM Multiscale Modeling and Simulation
Subjects:	**Pattern Formation and Solitons (nlin.PS)**; Soft Condensed Matter (cond-mat.soft); Optics (physics.optics)
Cite as:	**arXiv:1002.1986v2 [nlin.PS]**

Submission history

From: Boaz Ilan [view email]
[v1] Tue, 9 Feb 2010 21:52:33 GMT (91kb)
[v2] Thu, 18 Feb 2010 01:54:08 GMT (149kb)

Which authors of this paper are endorsers?