Connecting period-doubling cascades to chaos

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The appearance of infinitely-many period-doubling cascades is one of the most prominent features observed in the study of maps depending on a parameter. They are associated with chaotic behavior, since bifurcation diagrams of a map with a parameter often reveal a complicated intermingling of period-doubling cascades and chaos. Period doubling can be studied at three levels of complexity. The first is an individual period-doubling bifurcation. The second is an infinite collection of period doublings that are connected together by periodic orbits in a pattern called a cascade. It was first described by Myrberg and later in more detail by Feigenbaum. The third involves infinitely many cascades and a parameter value μ_2 of the map at which there is chaos. We show that often virtually all (i.e., all but finitely many) ``regular'' periodic orbits at μ_2 are each connected to exactly one cascade by a path of regular periodic orbits; and virtually all cascades are either paired -- connected to exactly one other cascade, or solitary -- connected to exactly one regular periodic orbit at μ_2. The solitary cascades are robust to large perturbations. Hence the investigation of infinitely many cascades is essentially reduced to studying the regular periodic orbits of $F(\mu_2, \cdot)$. Examples discussed include the forced-damped pendulum and the double-well Duffing equation.

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