On Decoding Irregular Tanner Codes with Local-Optimality Guarantees

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(Submitted on 13 Jul 2011 (v1), last revised 27 Nov 2011 (this version, v2))

We deal with decoding of Tanner codes using message-passing iterative decoding and linear programming (LP) decoding in memoryless binary-input output-symmetric (MBIOS) channels. We present a new combinatorial characterization for local-optimality of a codeword in irregular Tanner codes with respect to any MBIOS channel. This characterization is a generalization of [Arora, Daskalakis, Steurer; 2009] and [Vontobel; 2010] and is based on a conical combination of subtrees in the computation trees. The main novelty is that the subtrees may have any finite height h (even greater than the girth of the Tanner graph). In addition, the degrees of local-code nodes are not restricted to two. We prove that local-optimality in this new characterization implies Maximum-Likelihood (ML) optimality and LP-optimality. Given a codeword and the channel output, we also show how to efficiently recognize if the codeword is locally optimal.

We present a novel message-passing iterative decoding algorithm, called normalized weighted min-sum (NWMS). NWMS algorithm is a BP-type algorithm that applies to any Tanner code with single parity-check local codes (e.g., LDPC codes). We prove that if a locally optimal codeword for depth h exists, then the NWMS algorithm finds it in h iterations. Hence, the NWMS algorithm has an ML-certificate for any bounded number of iterations. Furthermore, since the depth h is not bounded, the guarantee for successful decoding by NWMS holds even if the number of iterations h exceeds the girth of the Tanner graph.

Finally, we apply the new local-optimality characterization to regular Tanner codes, and prove lower bounds on the noise thresholds of LP-decoding in MBIOS channels. When the noise is below these lower bounds, the probability that LP-decoding fails decays doubly exponentially in the girth of the Tanner graph.

Comments:	Submitted to IEEE Transactions on Information Theory
Subjects:	**Information Theory (cs.IT)**; Combinatorics (math.CO)
Cite as:	**arXiv:1107.2677 [cs.IT]**
	(or **arXiv:1107.2677v2 [cs.IT]** for this version)