



# The Bethe Permanent of a Non-Negative Matrix

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It has recently been observed that the permanent of a non-negative square matrix, i.e., of a square matrix containing only non-negative real entries, can very well be approximated by solving a certain Bethe free energy function minimization problem with the help of the sum-product algorithm. We call the resulting approximation of the permanent the Bethe permanent. In this paper we give reasons why this approach to approximating the permanent works well. Namely, we show that the Bethe free energy function is convex and that the sum-product algorithm finds its minimum efficiently. We then discuss the fact that the permanent is lower bounded by the Bethe permanent, and we comment on potential upper bounds on the permanent based on the Bethe permanent. We also present a combinatorial characterization of the Bethe permanent in terms of permanents of so-called lifted versions of the matrix under consideration. Moreover, we comment on possibilities to modify the Bethe permanent so that it approximates the permanent even better, and we conclude the paper with some observations and conjectures about permanent-based pseudo-codewords and permanent-based kernels.

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