Sparse approximation property and stable recovery of sparse signals from noisy measurements

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In this paper, we introduce a sparse approximation property of order s for a measurement matrix ${\bf A}$: $$\|{\bf x}_s\|_2\le D \|{\bf A}{\bf x}\|_2+ \beta \frac{\sigma_s({\bf x})}{\sqrt{s}} \quad {\rm for\ all} \ {\bf x},$$ where ${\bf x}_s$ is the best s-sparse approximation of the vector ${\bf x}$ in ℓ^2, $\sigma_s({\bf x})$ is the s-sparse approximation error of the vector ${\bf x}$ in ℓ^1, and D and β are positive constants. The sparse approximation property for a measurement matrix can be thought of as a weaker version of its restricted isometry property and a stronger version of its null space property. In this paper, we show that the sparse approximation property is an appropriate condition on a measurement matrix to consider stable recovery of any compressible signal from its noisy measurements. In particular, we show that any compressible signalcan be stably recovered from its noisy measurements via solving an ℓ^1-minimization problem if the measurement matrix has the sparse approximation property with $\beta\in (0,1)$, and conversely the measurement matrix has the sparse approximation property with $\beta\in (0,\infty)$ if any compressible signal can be stably recovered from its noisy measurements via solving an ℓ^1-minimization problem.

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