

Explicit Bounds for Entropy Concentration under Linear Constraints

Kostas N. Oikonomou

(Submitted on 29 Jul 2011 (v1), last revised 4 Oct 2011 (this version, v2))

Consider the construction of an object composed of m parts by distributing n units to those parts. For example, say we are assigning n balls to m boxes. Each assignment results in a certain count vector, specifying the number of balls allocated to each box. If only assignments satisfying a set of constraints that are linear in these counts are allowable, and m is fixed while n increases, most assignments that satisfy the constraints result in frequency vectors (normalized counts) whose entropy approaches that of the maximum entropy vector satisfying the constraints. This phenomenon of "entropy concentration" is known in various forms, and is one of the justifications of the maximum entropy method, one of the most powerful tools for solving problems with incomplete information. The appeal of entropy concentration comes from the simplicity of the argument: it is based purely on counting. Existing proofs of the concentration phenomenon are based on limits or asymptotics. Here we present non-asymptotic, explicit lower bounds on n for a number of variants of the concentration result to hold to any prescribed accuracies, taking into account the fact that allocations of discrete units can satisfy constraints only approximately. The results are illustrated with examples on die tossing, vehicle or network traffic, and the probability distribution of the length of a $G / G / 1$ queue.

Comments: Small addition in abstract. New paragraph on expectations vs. measurements in Intro between eqs. (1.1) and (1.2). In caption of Fig. 1.1, 23% -> 13.5%. Eliminated equation numbers in Lemmas 3.2 and 3.9, so now equations are re-numbered throughout. Added to the Conclusion. Added remarks after Lemmas 3.2 and 3.4 on the savings achieved by using Prop. 3.1

Subjects: **Information Theory (cs.IT)**; Data Analysis, Statistics and Probability (physics.data-an)

Cite as: [arXiv:1107.6004](https://arxiv.org/abs/1107.6004) [cs.IT]
(or [arXiv:1107.6004v2](https://arxiv.org/abs/1107.6004v2) [cs.IT] for this version)

Submission history

From: Kostas N. Oikonomou [[view email](mailto:k.oikonomou@cornell.edu)]

[v1] Fri, 29 Jul 2011 16:04:40 GMT (257kb)

[v2] Tue, 4 Oct 2011 18:57:50 GMT (257kb)

Which authors of this paper are endorsers?

Download:

- [PDF](#)
- [PostScript](#)
- [Other formats](#)

Current browse context:

cs.IT

[< prev](#) | [next >](#)

[new](#) | [recent](#) | [1107](#)

Change to browse by:

[cs](#)

[math](#)

[physics](#)

[physics.data-an](#)

References & Citations:

- [NASA ADS](#)

DBLP - CS Bibliography:

[listing](#) | [bibtext](#)

[Kostas N. Oikonomou](#)

Bookmark (what is this?)

