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Kalman R E. Contributions to the theory of optimal control.
Boletin de la Sociedad Matematica Mexicana **5**:102-19, 1960.
[Research Institute for Advanced Studies (Martin Marietta Corporation), Baltimore, MD]

The purpose of the paper is to treat dynamic optimization problems from a system-theoretic point of view, using the state-variable description of linear systems, the Hamilton-Jacobi formalism of the calculus of variations and, above all, the new concepts of *controllability* and *observability*. [The *SCI*® indicates that this paper has been cited over 175 times since 1961.]

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"It is most flattering and gratifying for a nonpharmacologist, nonbiologist, nonchemist, and nonstatistician, who is at the same time a non-know-how kind of theoretician, to have his paper become a 'Citation Classic' But a system-analysis study of the situation would have shown, a priori, that there were a number of reasons to expect just that.

"(1) This was my first comprehensive exposition of my 'modern' theory of linear dynamical systems, begun in 1954. It eventually combined such ideas as the state-variable description of systems, the basic insights concerning controllability and observability (controllability was later replaced by the more convenient notion of reachability), the duality principle, quadratic optimization via the Riccati equation, unified formalism for the time-dependent case, rigorous treatment of the infinite-horizon problem and the stability of the optimal system, consistent use of vector-space norms for estimating Lyapunov functions, and a brief mention of the (Penrose) generalized inverse.

"(2) The paper was published (as part of a conference proceedings in honor of Solomon Lefschetz) in a super-obscure Mexican journal which most libraries don't have and where the author is the only source of reprints. "(3) The paper contains some of those inessential minor mistakes (e.g., see the bottom of page 114) which encourage young lions to bite the tail of an older one

"(4) The conference having been scheduled just before my wedding (or vice versa), the paper was written in great haste but tried to give a full survey of my ideas during the preceding five years –hence a less unctuous style and more content than usual at ceremonial conferences

"(5) The paper introduced the notation (F, G, H) for a linear system (see equations (2.1-2 2)) which is today the worldwide standard.

"(6) The paper proceeds in a leisurely didactic style to help make the material accessible to persons with a moderate mathematical background

"The paper is still popular in 1979 because it is extremely difficult to find another problem area in system theory where such a complete mathematical treatment is possible with little more than linear algebra and elementary analysis plus a certain amount of awe and affection for the great men of the past Until, if ever, someone finds another chunk of systems and packages it just as successfully, the paper is bound to retain its appeal."