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Comment on “Phase Diagram of a Disordered Boson Hubbard Model in Two Dimensions”

In a recent Letter [1] (see also [2]) the authors presented numerical evidence supporting an idea of a direct transition between the superfluid (SF) and Mott insulating (MI) phases in the disordered Bosonic system, and even studied non-trivial properties of the multicritical line where SF, MI and the Bose Glass (BG) phases meet. The results were obtained from Monte Carlo simulations of the (2+1)-dimensional classical loop-current model [3] with the lattice action

$$S = \frac{1}{2K} \sum_{\mathbf{r}\tau}^{\nabla \cdot \vec{J}=0} \left[\vec{J}^2(\mathbf{r}, \tau) - 2(\mu + v(\mathbf{r})) \vec{J}_\tau(\mathbf{r}, \tau) \right]. \quad (1)$$

where $\mathbf{r}, \tau$ are spatial and imaginary time coordinates, and $\vec{J}(\mathbf{r}, \tau)$ are integer current vectors with zero divergence. The spatial disorder potential $v(\mathbf{r})$ is uniformly distributed on the interval $(-\Delta, \Delta)$.

Here we prove that all of the above mentioned conclusions are incorrect and originate from doing simulations for too small system sizes (the maximum system size considered in [1] was $L_x \times L_y \times L_\tau = 14 \times 14 \times 20$), and ignoring the rigorous theorem of [4, 5] saying that for disorder Δ larger than the half-width of the energy gap in the ideal Mott insulator (we denote it as E_g) the system state is compressible, i.e. it is BG. Indeed, in the infinite system one can always find arbitrary large regions with the chemical potential being nearly homogeneously shifted downwards or upwards by Δ . There is no energy gap then for the particle transfer between such regions, and they can be doped with particles/holes. [The conjecture is that the MI-BG transition is exactly at the upper boundary of the theorem, $\Delta = E_g(K)$.] We note, that the theorem is based on rare statistical fluctuations, and for $\Delta \rightarrow 0$ the distance between regions contributing to non-zero but *exponentially* small compressibility is diverging *exponentially*.

In Fig. 1 we plot our data for $E_g(K)$ [6] along with the critical values of disorder for the superfluid-insulator transition. Since Δ is always *larger* than $E_g(K_c)$, the transition is always from SF to the compressible insulating phase, or BG. Accordingly, the multicritical line where SF, MI, and BG phases meet does not exist for non-zero Δ .

The diverging distance between rare statistical realizations of disorder is the major problem in interpreting

Monte Carlo data for small system sizes. Recently developed Worm algorithms [6, 7] allow simulations of system sizes as large as $160 \times 160 \times 1000$, but for $\Delta = 0.4$ even this is barely enough to resolve very small but finite compressibility at the critical point [6]. Clearly, for smaller system sizes and smaller values of disorder the short length-scale behavior will mimic a direct SF-MI transition observed in Refs. [1, 2]. Similar arguments were used previously in the criticism of the direct SF-MI transition in one dimensional systems [8].

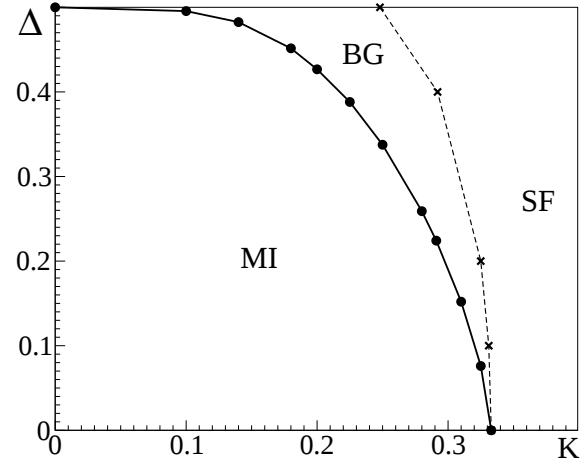


FIG. 1: The phase diagram of the disordered, commensurate loop-current model (1). All error bars are of order 10^{-3} and smaller than point sizes; data points for the SF-BG line were taken from: $\Delta = 0.4$ [1, 2, 6], $\Delta = 0.2$ [2], $\Delta = 0.1, 0.5$ [this work].

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