

## Mathematical Physics

# The Fourier $U(2)$ Group and Separation of Discrete Variables

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The linear canonical transformations of geometric optics on two-dimensional screens form the group  $Sp(4, \mathbb{R})$ , whose maximal compact subgroup is the Fourier group  $U(2)_F$ ; this includes isotropic and anisotropic Fourier transforms, screen rotations and gyrations in the phase space of ray positions and optical momenta. Deforming classical optics into a Hamiltonian system whose positions and momenta range over a finite set of values, leads us to the finite oscillator model, which is ruled by the Lie algebra  $so(4)$ . Two distinct subalgebra chains are used to model arrays of  $N^2$  points placed along Cartesian or polar (radius and angle) coordinates, thus realizing one case of separation in two discrete coordinates. The  $N^2$ -vectors in this space are digital (pixellated) images on either of these two grids, related by a unitary transformation. Here we examine the unitary action of the analogue Fourier group on such images, whose rotations are particularly visible.

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