



Mathematical Physics

A recursive approach to the $O(n)$ model on random maps via nested loops

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We consider the $O(n)$ loop model on tetravalent maps and show how to rephrase it into a model of bipartite maps without loops. This follows from a combinatorial decomposition that consists in cutting the $O(n)$ model configurations along their loops so that each elementary piece is a map that may have arbitrary even face degrees. In the induced statistics, these maps are drawn according to a Boltzmann distribution whose parameters (the face weights) are determined by a fixed point condition. In particular, we show that the dense and dilute critical points of the $O(n)$ model correspond to bipartite maps with large faces (i.e. whose degree distribution has a fat tail). The re-expression of the fixed point condition in terms of linear integral equations allows us to explore the phase diagram of the model. In particular, we determine this phase diagram exactly for the simplest version of the model where the loops are "rigid". Several generalizations of the model are discussed.

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