Mathematical Physics

Minimal length in quantum space and integrations of the line element in Noncommutative Geometry

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We question the emergence of a minimal length in quantum spacetime, comparing two notions that appeared at various points in the literature: on the one side, the quantum length as the spectrum of an operator L in the Doplicher Fredenhagen Roberts (DFR) quantum spacetime, as well as in the canonical noncommutative spacetime; on the other side, Connes' spectral distance in noncommutative geometry. Although on the Euclidean space the two notions merge into the one of geodesic distance, they yield distinct results in the noncommutative framework. In particular on the Moyal plane, the quantum length is bounded above from zero while the spectral distance can take any real positive value, including infinity. We show how to solve this discrepancy by doubling the spectral triple. This leads us to introduce a modified quantum length d'_L, which coincides exactly with the spectral distance d_D on the set of states of optimal localization. On the set of eigenstates of the quantum harmonic oscillator - together with their translations - d'_L and d_D coincide asymptotically, both in the high energy and large translation limits. At small energy, we interpret the discrepancy between d'_L and d_D as two distinct ways of integrating the line element on a quantum space. This leads us to propose an equation for a geodesic on the Moyal plane.

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