Mathematical Physics

Electromagnetism on Anisotropic Fractals

Martin Ostoja-Starzewski

(Submitted on 8 Jun 2011)

We derive basic equations of electromagnetic fields in fractal media which are specified by three indepedent fractal dimensions {\alpha}_{i} in the respective directions x_{i} (i=1,2,3) of the Cartesian space in which the fractal is embedded. To grasp the generally anisotropic structure of a fractal, we employ the product measure, so that the global forms of governing equations may be cast in forms involving conventional (integer-order) integrals, while the local forms are expressed through partial differential equations with derivatives of integer order but containing coefficients involving the {\alpha}_{i}'s. First, a formulation based on product measures is shown to satisfy the four basic identities of vector calculus. This allows a generalization of the Green-Gauss and Stokes theorems as well as the charge conservation equation on anisotropic fractals. Then, pursuing the conceptual approach, we derive the Faraday and Amp\`ere laws for such fractal media, which, along with two auxiliary null-divergence conditions, effectively give the modified Maxwell equations. Proceeding on a separate track, we employ a variational principle for electromagnetic fields, appropriately adapted to fractal media, to independently derive the same forms of these two laws. It is next found that the parabolic (for a conducting medium) and the hyperbolic (for a dielectric medium) equations involve modified gradient operators, while the Poynting vector has the same form as in the non-fractal case. Finally, Maxwell's electromagnetic stress tensor is reformulated for fractal systems. In all the cases, the derived equations for fractal media depend explicitly on fractal dimensions and reduce to conventional forms for continuous media with Euclidean geometries upon setting the dimensions to integers.

Subjects: **Mathematical Physics (math-ph)**
Cite as: **arXiv:1106.1491 [math-ph]**
(or **arXiv:1106.1491v1 [math-ph]** for this version)

Submission history

From: Martin Ostoja-Starzewski [view email]
[v1] Wed, 8 Jun 2011 03:37:43 GMT (10kb)