Mathematical Physics

Division Algebras, Supersymmetry and Higher Gauge Theory

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From the four normed division algebras--the real numbers, complex numbers, quaternions and octonions, of dimension k=1, 2, 4 and 8, respectively--a systematic procedure gives a 3-cocycle on the Poincare superalgebra in dimensions k+2=3, 4, 6 and 10, and a 4-cocycle on the Poincare superalgebra in dimensions k+3=4, 5, 7 and 11. The existence of these cocycles follow from spinor identities that hold only in these dimensions, and which are closely related to the existence of the superstring in dimensions k+2, and the super-2-brane in dimensions k+3.
In general, an (n+1)-cocycle on a Lie superalgebra yields a `Lie n-superalgebra': that is, roughly, an n-term chain complex equipped with a bracket satisfying the axioms of a Lie superalgebra up to chain homotopy. We thus obtain Lie 2-superalgebras extending the Poincare superalgebra in dimensions k+2, and Lie 3-superalgebras extending the Poincare superalgebra in dimensions k+3. We present evidence, based on the work of Sati, Schreiber and Stasheff, that these Lie n-superalgebras describe infinitesimal `higher symmetries' of the superstring and 2-brane.
Generically, integrating a Lie n-superalgebra to a Lie n-supergroup yields a `Lie n-supergroup' that is hugely infinite-dimensional. However, when the Lie n-superalgebra is obtained from an (n+1)-cocycle on a nilpotent Lie superalgebra, there is a geometric procedure to integrate the cocycle to one on the corresponding nilpotent Lie supergroup.
In general, a smooth (n+1)-cocycle on a supergroup yields a `Lie n-supergroup': that is, a weak n-group internal to supermanifolds. Using our geometric procedure to integrate the 3-cocycle in dimensions k+2, we obtain a Lie 2-supergroup extending the Poincare supergroup in those dimensions, and similarly integrating the 4-cocycle in dimensions k+3, we obtain a Lie 3-supergroup extending the Poincare supergroup in those dimensions.

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