Mathematical Physics

Approximate action-angle variables for the figure-eight and other periodic three-body orbits

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We use the maximally permutation symmetric set of three-body coordinates, that consist of the "hyper-radius" $R = \sqrt{\rho^{2} + \lambda^{2}}$, the "rescaled area of the triangle" $\frac{\sqrt 3}{2 R^2} |{\bm \rho} \times {\bm \lambda}|$) and the (braiding) hyper-angle $\phi = \arctan(\frac{2{\bm \rho} \cdot {\bm \lambda}}{\lambda^2 - \rho^2})$, to analyze the "figure-eight" choreographic three-body motion discovered by Moore \cite{Moore1993} in the Newtonian three-body problem. Here ${\bm \rho}, {\bm \lambda}$ are the two Jacobi relative coordinate vectors. We show that the periodicity of this motion is closely related to the braiding hyper-angle ϕ. We construct an approximate integral of motion ${\bar{G}}$ that together with the hyper-angle ϕ forms the action-angle pair of variables for this problem and show that it is the underlying cause of figure-eight motion's stability. We construct figure-eight orbits in two other attractive permutation-symmetric three-body potentials. We compare the figure-eight orbits in these three potentials and discuss their generic features, as well as their differences. We apply these variables to two new periodic, but non-choreographic orbits: One has a continuously rising ϕ in time t, just like the figure-eight motion, but with a different, more complex periodicity, whereas the other one has an oscillating $\phi (t)$ temporal behavior.

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