

## Mathematical Physics

# Series representations of the Riemann and Hurwitz zeta functions and series and integral representations of the first Stieltjes constant

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We develop series representations for the Hurwitz and Riemann zeta functions in terms of generalized Bernoulli numbers (Nörlund polynomials), that give the analytic continuation of these functions to the entire complex plane. Special cases yield series representations of a wide variety of special functions and numbers, including log Gamma, the digamma, and polygamma functions. A further byproduct is that  $\zeta(n)$  values emerge as nonlinear Euler sums in terms of generalized harmonic numbers. We additionally obtain series and integral representations of the first Stieltjes constant  $\gamma_1(a)$ . The presentation unifies some earlier results.

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