Mathematical Physics

Spectral properties of a 2D scalar wave equation with 1D-periodic coefficients: application to SH elastic waves

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The paper provides a rigorous analysis of the dispersion spectrum of SH (shear horizontal) elastic waves in periodically stratified solids. The problem consists of an ordinary differential wave equation with periodic coefficients, which involves two free parameters $\omega $ (the frequency) and k (the wavenumber in the direction orthogonal to the axis of periodicity). Solutions of this equation satisfy a quasi-periodic boundary condition which yields the Floquet parameter K. The resulting dispersion surface $\omega (K,k)$ may be characterized through its cuts at constant values of K, k and $\omega $ that define the passband (real K) and stopband areas, the Floquet branches and the isofrequency curves, respectively. The paper combines complementary approaches based on eigenvalue problems and on the monodromy matrix $\mathbf{M}$. The pivotal object is the Lyapunov function $\Delta (\omega ^{2},k^{2}) \equiv 1/2\mathrm{trace}\mathbf{M}=\cos K$ which is generalized as a function of two variables. Its analytical properties, asymptotics and bounds are examined and an explicit form of its derivatives obtained. Attention is given to the special case of a zero-width stopband. These ingredients are used to analyze the cuts of the surface $\omega (K,k).$ The derivatives of the functions $\omega (k)$ at fixed K and $\omega (K)$ at fixed k and of the function $K(k)$ at fixed $\omega $ are described in detail. The curves $\omega (k)$ at fixed K are shown to be monotonic for real $K,$ while they may be looped for complex K (i.e. in the stopband areas). The convexity of the closed (first) real isofrequency curve $K(k)$ is proved thus ruling out low-frequency caustics of group velocity. The results are relevant to the broad area of applicability of ordinary differential equation for scalar waves in 1D phononic (solid or fluid) and photonic crystals.

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