



# Time-Reversal Symmetry in Non-Hermitian Systems

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For ordinary hermitian Hamiltonians, the states show the Kramers degeneracy when the system has a half-odd-integer spin and the time reversal operator obeys  $\Theta^2 = -1$ , but no such a degeneracy exists when  $\Theta^2 = +1$ . Here we point out that for non-hermitian systems, there exists a degeneracy similar to Kramers even when  $\Theta^2 = +1$ . It is found that the new degeneracy follows from the mathematical structure of split-quaternion, instead of quaternion from which the Kramers degeneracy follows in the usual hermitian cases. Furthermore, we also show that particle/hole symmetry gives rise to a pair of states with opposite energies on the basis of the split quaternion in a class of non-hermitian Hamiltonians. As concrete examples, we examine in detail  $N \times N$  Hamiltonians with  $N=2$  and  $4$  which are non-hermitian generalizations of spin  $1/2$  Hamiltonian and quadrupole Hamiltonian of spin  $3/2$ , respectively.

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