General Relativity and Quantum Cosmology

The helical phase of chiral nematic liquid crystals as the Bianchi VII(0) group manifold

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We show that the optical structure of the helical phase of a chiral nematic is naturally associated with the Bianchi VII(0) group manifold, of which we give a full account. The Joets-Ribotta metric governing propagation of the extraordinary rays is invariant under the simply transitive action of the universal cover of the three dimensional Euclidean group of two dimensions. Thus extraordinary light rays are geodesics of a left-invariant metric on this Bianchi type VII(0) group. We are able to solve by separation of variables both the wave equation and the Hamilton-Jacobi equation for this metric. The former reduces to Mathieu's equation and the later to the quadrantal pendulum equation. We discuss Maxwell's equations for uniaxial optical materials where the configuration is invariant under a group action and develop a formalism to take advantage of these symmetries. The material is not assumed to be impedance matched, thus going beyond the usual scope of transformation optics. We show that for a chiral nematic in its helical phase Maxwell's equations reduce to a generalised Mathieu equation. Our results may also be relevant to helical phases of some magnetic materials and to light propagation in certain cosmological models.

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