Unleashing the power of Schrijver's permanental inequality with the help of the Bethe Approximation

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Let $A \in \Omega_n$ be doubly-stochastic $n \times n$ matrix. Alexander Schrijver proved in 1998 the following remarkable inequality per(\widetilde{A}) \geq \prod_{1 \leq i,j \leq n} (1- A(i,j)); \widetilde{A}(i,j) =: A(i,j)(1-A(i,j)), 1 \leq i,j \leq n.
We use the above Shrijver's inequality to prove the following lower bound: \frac{per(A)}{F(A)} \geq 1; F(A) =: \prod_{1 \leq i,j \leq n} (1- A(i,j))^{1- A(i,j)}.
We use this new lower bound to prove S.Friedland's Asymptotic Lower Matching Conjecture(LAMC) on monomer-dimer problem.
We use some ideas of our proof of (LAMC) to disprove [Lu,Mohr,Szekely] positive correlation conjecture.
We present explicit doubly-stochastic $n \times n$ matrices A with the ratio $\frac{per(A)}{F(A)} = \sqrt{2}^{n}$; conjecture that \max_{A \in \Omega_n}\frac{per(A)}{F(A)} \approx (\sqrt{2})^{n} and give some examples supporting the conjecture.
If true, the conjecture (and other ones stated in the paper) would imply a deterministic poly-time algorithm to approximate the permanent of $n \times n$ nonnegative matrices within the relative factor $(\sqrt{2})^{n}$. The best current such factor is e^n.

Comments:	30 pages, more typos are fixed, more remarks are added, importantly a concrete counter-example to [Lu,Mohr,Szekely] positive correlation conjecture is presented
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