Mathematics > Dynamical Systems

On the general one-dimensional XY Model: positive and zero temperature, selection and non-selection

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We consider (M,d) a connected and compact manifold and we denote by $\mathcal{B}_i$ the Bernoulli space $M^{\Z}$ of sequences represented by $$x=(... x_{-3},x_{-2},x_{-1},x_0,x_1,x_2,x_3,...),$$ where x_i belongs to the space (alphabet) M. The case where $M=\mathbb{S}^1$, the unit circle, is of particular interest here. The analogous problem in the one-dimensional lattice $\mathbb{N}$ is also considered. %In this case we consider the potential $A: {\cal B}=M^\mathbb{N} \to \mathbb{R}.$ Let $A: \mathcal{B}_i \rar \R$ be an {\it observable} or {\it potential} defined in the Bernoulli space $\mathcal{B}_i$. The potential A describes an interaction between sites in the one-dimensional lattice $M^\mathbb{Z}$. Given a temperature T, we analyze the main properties of the Gibbs state $\hat{\mu}_{\frac{1}{T} A}$ which is a certain probability measure over ${\cal B}_i$. We denote this setting "the general XY model". In order to do our analysis we consider the Ruelle operator associated to $\frac{1}{T} A$, and, we get in this procedure the main eigenfunction $\psi_{\frac{1}{T} A}$. Later, we analyze selection problems when temperature goes to zero: a) existence, or not, of the limit (on the uniform convergence) $$V:=\lim_{T\to 0} T\, \log(\psi_{\frac{1}{T} A}),\,\,\,\,\text{a question about selection of subaction},$$ and, b) existence, or not, of the limit (on the weak* sense) $$\tilde{\mu}:=\lim_{T\to 0} \hat{\mu}_{\frac{1}{T}\, A},\,\,\,\,\text{a question about selection of measure}.$$ The existence of subactions and other properties of Ergodic Optimization are also considered.

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