Classical Bethe Ansatz and Normal Forms in the Jaynes-Cummings Model

O. Babelon, B. Doucot

(Submitted on 16 Jun 2011)

The Jaynes-Cummings-Gaudin model describes a collection of n spins coupled to an harmonic oscillator. It is known to be integrable, so one can define a moment map which associates to each point in phase-space the list of values of the $n+1$ conserved Hamiltonians. We identify all the critical points of this map and we compute the corresponding quadratic normal forms, using the Lax matrix representation of the model. The normal coordinates are constructed by a procedure which appears as a classical version of the Bethe Ansatz used to solve the quantum model. We show that only elliptic or focus-focus singularities are present in this model, which provides an interesting example of a symplectic toric action with singularities. To explore these, we study in detail the degeneracies of the spectral curves for the $n=1$ and $n=2$ cases. This gives a complete picture for the image of the momentum map (IMM) and the associated bifurcation diagram. For $n=2$ we found in particular some lines of rank 1 which lie, for one part, on the boundary of the IMM, where they behave like an edge separating two faces, and which go, for another part, inside the IMM.

Comments:	39 pages, 14 figures
Subjects:	**Statistical Mechanics (cond-mat.stat-mech)**; Mathematical Physics (math-ph)
Cite as:	**arXiv:1106.3274 [cond-mat.stat-mech]**
	(or **arXiv:1106.3274v1 [cond-mat.stat-mech]** for this version)

Submission history

From: Benoit Doucot [view email]
[v1] Thu, 16 Jun 2011 17:00:13 GMT (890kb)