Mathematical Physics

Geometry and Shape of Minkowski's Space Conformal Infinity

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We review and further analyze Penrose's 'light cone at infinity' - the conformal closure of Minkowski space. Examples of a potential confusion in the existing literature about it's geometry and shape are pointed out. It is argued that it is better to think about conformal infinity as of a needle horn supercyclide (or a limit horn torus) made of a family of circles, all intersecting at one and only one point, rather than that of a 'cone'. A parametrization using circular null geodesics is given. Compactified Minkowski space is represented in three ways: as a group manifold of the unitary group U(2) a projective quadric in six-dimensional real space of signature (4,2) and as the Grassmannian of maximal totally isotropic subspaces in complex four--dimensional twistor space. Explicit relations between these representations are given, using a concrete representation of antilinear action of the conformal Clifford algebra Cl(4,2) on twistors. Concepts of space-time geometry are explicitly linked to those of Lie sphere geometry. In particular conformal infinity is faithfully represented by planes in 3D real space plus the infinity point. Closed null geodesics trapped at infinity are represented by parallel plane fronts (plus infinity point). A version of the projective quadric in six-dimensional space where the quotient is taken by positive reals is shown to lead to a symmetric Dupin's type `needle horn cyclide' shape of conformal infinity.

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