Mathematical Physics

Critical partitions and nodal deficiency of billiard eigenfunctions

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The paper addresses the the number of nodal domains for eigenfunctions of Schr\"{o}dinger operators with Dirichlet boundary conditions in bounded domains. In dimension one, the nth eigenfunction has n nodal domains. The Courant Theorem claims that in any dimension, the number of nodal domains of the nth eigenfunction cannot exceed n. However, in dimensions higher than 1 the equality can hold for only finitely many eigenfunctions. Thus, a "nodal deficiency" arises. Examples are known of eigenfunctions with arbitrarily large index n that have just two nodal domains.

It was suggested in the recent years to look at the partitions of the domain, rather than eigenfunctions. It was shown in a recent paper by Helffer, Hoffmann-Ostenhof and Terracini that (under some natural conditions) bipartite partitions minimizing the maximum of the ground-state energies in sub-domains of the partition, correspond to the "Courant sharp" eigenfunctions, i.e. to those with zero nodal deficiency.

In this paper, the authors show, under some genericity conditions, among the bipartite equipartitions, the nodal ones correspond exactly to the critical points of an analogous functional, with the nodal deficiency being equal to the Morse index at this point. This explains, in particular, why all the minimal partitions must be Courant sharp.

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