Mathematical Physics

On the Quasi-Linear Elliptic PDE $-\nabla\cdot(\nabla{u}/\sqrt{1-|\nabla{u}|^2}) = 4π\sum_k a_k δ_{s_k}$ in Physics and Geometry

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It is shown that for each finite number of Dirac measures supported at points s_n in three-dimensional Euclidean space, with given amplitudes a_n, there exists a unique real-valued Lipschitz function u, vanishing at infinity, which distributionally solves the quasi-linear elliptic partial differential equation of divergence form $-\nabla\cdot(\nabla{u}/\sqrt{1-|\nabla{u}|^2})=4\pi\sum_{n=1}^N a_n \delta_{s_n}$. Moreover, u is real analytic away from the s_n. The result can be interpreted in at least two ways: (a) for any number of point charges of arbitrary magnitude and sign at prescribed locations s_n in three-dimensional Euclidean space there exists a unique electrostatic field which satisfies the Maxwell-Born-Infeld field equations smoothly away from the point charges and vanishes as $|s|\to\infty$; (b) for any number of integral mean curvatures assigned to locations s_n there exists a unique asymptotically flat, almost everywhere space-like maximal slice with point defects of Minkowski spacetime, having lightcone singularities over the s_n but being smooth otherwise, and whose height function vanishes as $|s|\to\infty$. No struts between the point singularities ever occur.

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