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Research Article

The Meaning of Time and Covariant Superderivatives in Supermechanics

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Abstract

We present a review of the basics of supermanifold theory (in t
physicist's point of view. By considering a detailed example of w
ordinary superdifferential equation" we show how the appearanc
time is very natural in this context. We conclude that in dynar
supermanifolds, the space that classically parametrizes time (the r
supermanifold □1|1. This supermanifold admits several different
group-theoretic point of view what is the meaning of the usual co
in the underlying group law. This result is extended to the case of l

1. Introduction

The usual interpretation of time in physics (at least in classical me
described by a Hamiltonian H, which is a differentiable function de
trajectories are then identified with integral curves of a vector
curves define a flow, that is, a differentiable mapping $\Phi : I \times M \to$
maximal integral curve passing through $p \in M$ at $t = 0$ and $I \subset$ I

$\{c_p\}_{p\in M}$ of integral curves. Then, “time” is the space of values manifold structure. In some cases (e.g., dynamics on compact ma Lie group with the operation of addition although, in general, this c

Our goal in this paper is to describe in analog terms what shoul dynamical system implementing some kind of supersymmetry or system represented by a supervector field on a supermanifold.

Let us say, in advance, that the answer is not new. The so-called ago, but its introduction almost always has been a matter of estl has to give a “super” partner of the bosonic time t) or an *ac* observed that the index formula could be understood in terms of which the parameter space is given by pairs (t,θ), with t bosonic complete proof of this result using this “supertime.” The paper [3 setup of classical mechanics viewed as a field theory in one time a G. O. Freund's book [16, see Chapter 10]. Also, a supertime (quantization model for the superparticle (a detailed account can b convenient construct, but not a necessary geometric object wh structure of the theory. Indeed, it is not unusual to find in ph classical geometric constructions such as geodesics, normal coord which the parameter is just the common bosonic time t. Also, ther the form of a single odd parameter (as in [6]) and others with a s even one t, these models being referred to as $N=2$ supersymmetr supersymmetric models with $N\geq 2$ (see, e.g., [10]). As an altern supermanifolds (in the sense we will consider in these notes, fo Lagrangian and Hamiltonian viewpoints, we refer the reader to [11

Thus, in view of this variety of constructions, it seems interesting analog of “time” (as understood in classical mechanics) should b exposed above, and we will try to show why it is necessary to int theories on supermanifolds by means of some examples (which, later). This is a consequence of a profound result due to J. Mor seems to be not very well known by physicists working with supe and proof. The result is a completely general theorem of exis equations on supermanifolds. It gives the most general answer in existence of solutions for an arbitrary supervector field.

In the second part of the paper, we analyze the construction o relationship with supertime. Also, we consider their meaning withi superalgebras, showing how their introduction amounts to a redefi supertime.

The theory of superdifferential equations was initially studied b outside the circle of soviet mathematicians that deserves much classes of supervector fields that can be integrated and classified however, was incomplete and superseded by the treatment in [13 how to deal with the problem of integrating supervector fields. Bo also offered some particular examples considering the supermanif as its superfunctions are just differential forms. We will also work this way, we hope that we can explain the mathematical theory be

2. The Algebraic Language in Differential Geometry

Throughout this paper we assume that the reader has some fam geometry and manifold theory as presented in any of the nu physicists (see e.g.[15 - 18].) However, we do not assume any pre

In physics, it is usual to describe a system in terms of its obser system by making measurements on it, that is, by evaluating tl system. Now, how do we describe observables (in the classical -no

Think of a system composed by one single particle. Classically, to and momentum $\mathbf{p} = (p^1, p^2, p^3)$ (we are assuming that there are values). Thus, we need a $2 \cdot 3 = 6$-dimensional manifold M whose el is called the phase space of the system, and it usually has submanifold $Q \subset \mathbb{R}^3$. Then, an observable is a function on the phas is given by $T(\mathbf{x},\mathbf{p}) = (1/2m)\|\mathbf{p}\|^2$, where $\|\cdot\|$ denotes the norm or associated to the induced metric, and m is the mass of the particle

As the equations of classical mechanics are differential equations i of regularity for these functions; indeed, it is usual to take infinite observables is in turn of the type $C^\infty(M)$. What is the structure manifold? In short (see [19, 20] as advanced references), they h $U \subset M$ which is open, we have a subset $C^\infty(U) \subset C^\infty(M)$ in such a wa

(i) $C^\infty(U)$ is an algebra (where the sum and product are giv any $p \in M$ and $f, g \in C^\infty(U)$);

(ii) for each pair of open sets $V \subset U$ of M, there is defined a r

(1) $\rho^U_U = id_U$ for all $U \subset M$ open,

(2) whenever we have open sets $W \subset V \subset U$, $\rho^U_W = \rho^V_W \circ \rho^U_V$,

(3) if $U \subset M$ is open and $\{U_i\}_{i \in I}$ is an open covering $\rho^U_{U_i}(f) = \rho^U_{U_i}(g)$ (in other words, the restrictions $f|_{U_i}, g|_{U_i}$ to a

These properties are embodied in the mathematical notion $M \supset U \mapsto C^\infty(U) \subset C^\infty(M)$ turns $C^\infty(M)$ into a sheaf of (commutative) we want to stress this fact, we write C^∞_M instead of $C^\infty(M)$. Indeed, we are given a way to distinguish the sets $U \subset M$ which are open $C^\infty(M)$) with some mild properties (see [19]), the manifold structu is called the structural sheaf of the ordinary manifold M. What is hence "almost all" the physics on M) can be described in tern derived from it.

As an example, take a differentiable vector field X on M (this is de at a point p, X_p as an equivalence class of curves $c : \mathbb{R} \to M$ on the

equivalent at a point $p \in M$ if and only if $c_1(0) = p = c_2(0)$ and $(dc_1$
is the same as to give a mapping $X_p : C^{\infty}(M) \to \mathbb{R}$ such that if $f, g \in$

$$X_p(f \cdot g) = X_p(f) \cdot g(p) + f($$

(see [21]). The idea is to define, for a given $f \in C^{\infty}(M)$, the action c
a representant of the class X_p, and then to apply the chain
$X = X^i(\partial / \partial x^i)$, where $X^i \in C^{\infty}(M)$ (for $1 \le i \le n$) and $\partial / \partial x^i$ acts like th

$$\frac{\partial}{\partial x^i}(f \cdot g) = \frac{\partial f}{\partial x^i} \cdot g + f \cdot$$

Thus, we can characterize $\mathcal{X}(M)$ as the set of mappings $X : C^{\infty}(M)$ -
functions and products by scalars of $\mathbb{R}$ and, in addition, satisfy (2.1
identification, that we deal with the C^{∞} category. For C^r vector fi
reasons, this set is called the set of derivations of $C^{\infty}(M)$, and it is d

Actually, $\mathrm{Der}\, C^{\infty}(M)$ has a lot more of structure. It is a real vector s
as a $\mathbb{K}$-vector space but now $C^{\infty}(M)$ is a ring, not a field $\mathbb{K}$. Nev
module (akin to the dual V^* of a $\mathbb{K}$-vector space V) $\mathrm{Der}^*\, C^{\infty}(M)$, ar
1-forms $\mathrm{Der}^*\, C^{\infty}(M) = \Omega^1(M)$. From $\mathcal{X}(M)$ and $\Omega^1(M)$, by taking te
tensor field on M, and develop in this way all the usual concepts of

To summarize: what is really important to get a physical descri
observables are. Mathematically, this is reflected in the fact that
enough to say what are the differentiable functions on M, that is, t

3. Review of the Classical (Nongraded) Case

Consider first the problem of determining the integral curves of
coordinate system $\{x^i\}_{i=1}^n$ the vector field $X \in \mathcal{X}(M)$ has the
convention)

$$X = X^i \frac{\partial}{\partial x^i},$$

what we want is to find the curves $c : I_c \subset \mathbb{R} \to M$ satisfying

$$\frac{dc}{dt}(t) = X_{c(t)}, \quad \forall t$$

subject to an initial condition $c(t_0) = p$ and $(dc / dt)(t_0) = X_p$. Let u
integral curves are defined for all the values of the time paramete
the case, e.g., of compact manifolds) and take $t_0 = 0$. Under this a
of diffeomorphisms $\{\varphi_t\}_{t \in \mathbb{R}}$, where

$$\varphi_t : \mathrm{M} \to \mathrm{M}, \quad p \mapsto \varphi_t(p) = c_p$$

c_p being the integral curve of X such that $c_p(0) = p$. Moreover, the f

$$\varphi_{t+s} = \varphi_t \circ \varphi_s, \quad \forall t$$
$$\varphi_{-t} = (\varphi_t)^{-1}.$$

It is very important to note the converse: each time we have a (3.4), we can construct a vector field $X \in \mathcal{X}(M)$ associated to it as

$$X(p) = \frac{d}{dt}\bigg|_{t=0} (t \mapsto \varphi_t$$

Remark 3.1. Diffeomorphisms on M extend themselves to automo (vector bundles, exterior algebras, etc.). For instance, a diffeomor the algebra $C^{\infty}(M)$ by means of the action $\varphi(f) = f \circ \varphi^{-1}$ for any $f \in C$

We insist on the fact that if we have computed in some way a fam field (the infinitesimal generator) *by taking derivatives with respe* essential to later introduce the notion of "supertime," in Section

At this point, we want to stress yet another feature of this integr Lie group appears, so does its associated Lie algebra. In the assumption that $I_c = \mathbb{R}$), the parameter family forms a Lie group: trivial, being abelian unidimensional. We will see later, in Section possible Lie supergroup structures for the family of integrating par regarding the class of vector fields that can be integrated.

4. Differential Equations on Supermanifolds

Now consider the case of supermanifolds. F. Berezin was the first between bosons and fermions in a quantum mechanical system (s is due to J. Martin [23]), and to treat this operation as if it we formalism, encompassing both fermions and bosons, was possible object in the theory, those associated to fermions carrying degree The rules for operating with this degree were easily obtained from symmetry to the system, and they were found to correspond to modulo 2. Of course, observables are examples of what we unders if we want to allow from the start the possibility of having a symm fermion-boson interchange, we must assign a degree to each obs with the rest of algebraic structures that we have defined on C^{∞} ways to carry on that "$\mathbb{Z}_2$ extension," and we will consider her [24], D. Leites [25], and Y. I. Manin [26]. Alternative approaches general theory and the relations between these approaches in [30]

In this context, a supermanifold can be thought as an ordinary m functions C^{∞}_M on M (which is a sheaf of commutative algebras) h superalgebras $\mathcal{A}_M$, so now to each open set $U \subset M$ we will associ written $\mathcal{M} = (M, \mathcal{A}_M)$.

Remark 4.1. A superalgebra is simply a vector space $\mathcal{A}$ which h decomposition $\mathcal{A} = \mathcal{A}_0 \oplus \mathcal{A}_1$ (the factors are indexed by the eler decomposition, that is, a binary operation $\cdot : \mathcal{A} \times \mathcal{A} \to \mathcal{A}$ such that $\mathcal{A}$

$i = 0, 1$) are called homogeneous of degree i, and it is clear that an

homogeneous ones. If $v \in \mathcal{A}_i$ we write $|v| = i$ to express the degree

A superalgebra $\mathfrak{g}$ is a Lie superalgebra if its product $[,] : \mathfrak{g} \times \mathfrak{g} \to \mathfrak{g}$ sat

$$[x, y] = -(-1)^{|x||y|}[$$

and the graded Jacobi identity

$$(-1)^{|x||z|}[[x, y], z] + (-1)^{|y||x|}[[y, z], x]$$

These conditions are just generalizations of the usual ones chara "golden rule of signs:" $a \cdot b = (-1)^{|a||b|} b \cdot a$ (i.e., each time two el factor powered to the product of the degrees appear).

A more general setting consists in a sheaf of $\mathbb{Z}$-graded algebras. I and a product such that $A_k \cdot A_l \subset A_{k+l}$.

Every $\mathbb{Z}$-graded algebra can be turned into a superalgebra simply b $\mathcal{A}_0$ and those with an odd index in $\mathcal{A}_1$. That is, we write $\mathcal{A} = \mathcal{A}_0 \oplus \mathcal{A}$

The definition of supermanifold implies some features that are abs cannot think of $\mathcal{A}_U$ as an algebra of real functions (as these ar elements of $\mathcal{A}_U$ on points of M as is done for usual functions. This point, but even if this is the case, it should be specified how to r any other evaluation taking all the possible sets U containing the supermanifold as a set of points, we run into trouble when we valued functions): all of them must give zero when evaluated on has even part). Thus, it is useless to speak of "points" of a sup said that "a supermanifold does not have points." (However, it i such a way that local expressions of supervector fields, superforr (see [31]).)

We can develop a differential geometry in a supermanifold followi point is to keep in mind that all the constructions must be made fr supermanifold $\mathcal{M} = (M, \Omega_M)$, where $\Omega_M = \oplus_{k \in \mathbb{Z}} \Omega^k(M)$ are the diffe $\Omega^0(M) = C^\infty(M)$, and the product is given by the wedge product of consider the differential forms on U, $\oplus_{k \in \mathbb{Z}} \Omega^k(U)$, so we have a sh a sheaf of superalgebras (which is sometimes known as the Cart now as an element $\omega = \Sigma \omega_{(i)}$, where $\omega_{(i)} \in \Omega^i(M)$ are the homoge $i \geq 0$.

The simplest case is the supermanifold $(\mathbb{R}, \Omega(\mathbb{R}))$, also denoted by example because it also illustrates the notion of a Lie supergroup. by θ. Thus, note that superfunctions are now differential forms c differentiable functions $f, g : \mathbb{R} \to \mathbb{R}$. A classical theorem of Frölich algebra (such as $\Omega(\mathbb{R})$) can be expressed in the form $\mathcal{L}_K + i_J$ for so

our case, the base is $\mathbb{R}$ and $J = \partial / \partial t = K$, where t is the canonical
basis derivations on an element $f(t) + g(t)\theta$ is given by

$$\mathcal{L}_{\partial/\partial t}(f(t) + g(t)\theta) = \frac{\partial f}{\partial t}$$
$$i_{\partial/\partial t}(f(t) + g(t)\theta) =$$

Due to these formulae, we will write

$$\mathcal{L}_{\partial/\partial t} \equiv \frac{\partial}{\partial t}, \qquad i_{\partial/\partial t}$$

(Note that the degrees as $\mathbb{Z}_2$-graded endomorphisms are $|\partial / \partial t| = 0$

As we have mentioned, $\mathbb{R}^{1|1}$ admits several Lie supergroup struct
details), giving the result of the composition $(f_1(t) + g_1(t)\theta) * (f_2(t)$

Table 1: Lie supergroup structures on $\mathbb{R}^{1|1}$.

This is not the standard way of presenting these Lie supergrou
supercoordinates t and θ on $\mathbb{R}^{1|1}$, which acts as $t(f_1(t) + g_1(t)\theta) =$
written $a = (t(a), \theta(a))$ and we get now Table 2.

Table 2: Lie supergroup structures on $\mathbb{R}^{1|1}$.

What is the analog of a classical vector field in this setting? To cha
as derivations on the algebra $C^{\infty}(M)$ (recall (2.1). Thus, a vecto
derivation $V : \Omega_M \to \Omega_M$, and this means that V is a graded morp
degree $|V|$: $V(\Omega^k(M)) \subset \Omega^{k+|V|}(M)$), and it verifies Leibniz's rule:
examples of derivations on Ω_M are the exterior differential, d,
$X \in \mathcal{X}(M)$, $\mathcal{L}_X$, and the insertion i_X. Indeed, these operators consic
$\Omega(M)$ (with degrees $|\mathcal{L}_X| = 0$, $|i_X| = -1$, $|d| = 1$, for any $X \in \mathcal{X}(M)$) gene
product is the composition of endomorphisms and the Lie superbra
of graded endomorphisms: $[E, F] = E \circ F - (-1)^{|E||F|} F \circ E$, for all E,
computed by using Cartan calculus; for example

$$[\mathcal{L}_X, i_Y] = \mathcal{L}_X \circ i_Y - (-1)^{0 \cdot (-1)} i_Y \circ \mathcal{L}_X = \mathcal{L}$$

Now, we would like to know what does it mean to integrate such a
integrating the derivation given by the exterior differential $d : \Omega_M$
that $d(\Omega^i(M)) \subset \Omega^{i+1}(M)$ and

$$d(\alpha \wedge \beta) = d\alpha \wedge \beta + (-1)^{1 \cdot |\alpha|} \alpha \wedge d$$

By analogy with (3.3) we seek for a curve $\omega : \mathbb{R} \to \Omega_M$ (also denote

$$\frac{d\omega}{dt}(t) = d(\omega(t)), \quad \forall$$

To find a solution we may proceed formally and construct the expo

$$\omega(t) = e^{td}\omega_0,$$

$\omega_0 \in \Omega_M$ being the initial condition $\omega(0) = \omega_0$. Here we understar have any *a priori* notion of convergence for such a series. Of cours sense, but here an important property of the particular derivatio series in (4.8) reduces to a finite sum and, indeed,

$$\omega(t) = (I + t \cdot d)\omega($$

where $I \in \mathrm{End}\Omega_M$ is the identity morphism.

5. The Need for Supertime

Thus, it seems that we have solved the problem of integrating a s technical questions pending. Recall that in the classical case we (which in turn extend their action as automorphisms to all of $\Omega($ defined vector field on M from them. We should ask for a similar Indeed, if we consider $C^\infty(M)$ instead of $\Omega(M)$ (or the sheaf $\mathcal{A}$ in ger is a theorem implying that we must demand that the morphisms ϵ [33]): let E be a vector bundle over a manifold M, then, each au which, in turn, induces a diffeomorphism on M.

Now, let us check whether $\{e^{t\cdot d}\}_{t\in\mathbb{R}}$ is a family of automorphisms the one hand,

$$\begin{aligned} e^{t\cdot d}(\alpha \wedge \beta) &= (I + t\cdot d)(\alpha \wedge \beta) \\ &= \alpha \wedge \beta + t\cdot d(\alpha \wedge \beta) \\ &= \alpha \wedge \beta + t\cdot d\alpha \wedge \beta + \end{aligned}$$

and on the other,

$$\begin{aligned} e^{t\cdot d}\alpha \wedge e^{t\cdot d}\beta &= (\alpha + t\cdot d\alpha) \wedge (\beta + t\cdot d \\ &= \alpha \wedge \beta + t\cdot d\alpha \wedge \beta + t\cdot \end{aligned}$$

It is then clear that

$$e^{t\cdot d}(\alpha \wedge \beta) \neq e^{t\cdot d}\alpha \wedge$$

so $\{e^{t\cdot d}\}_{t\in\mathbb{R}}$ is *not* a family of automorphisms and we cannot clair

A comparison of (5.1) and (5.2) shows us the way out to this impa treating t as a formal parameter, not necessarily in $\mathbb{R}$, so we n elements of $\Omega(M)$. Repeating the computations we get

$$e^{t\cdot d}(\alpha \wedge \beta) = \alpha \wedge \beta + t\cdot d\alpha \wedge \beta + (\text{-}$$
$$e^{t\cdot d}\alpha \wedge e^{t\cdot d}\beta = \alpha \wedge \beta + t\cdot d\alpha \wedge \beta + \alpha$$

The problem here is that the term containing two instances of th sign is required to pass from $\alpha \wedge t\cdot d\beta$ to $(-1)^{|\alpha|} t\cdot\alpha \wedge d\beta$. These inco $t \in \mathbb{R}$, but $t = \theta$ an anticommuting parameter (so $\theta^2 = 0$) of $\mathbb{Z}$-degre is immediate that

$$e^{\theta\cdot d}(\alpha \wedge \beta) = e^{\theta\cdot d}\alpha \wedge$$

and the problem of integrating the vector field d on the superman sense that we integrate vector fields on a manifold: it admits a one

In Section 3 we saw that the differential equation is recovered by the family of integrating automorphisms, so we are led to conside note that if we write $\omega(t,\theta) = (I + \theta\cdot d)\omega_0$ for the solution that we ha

$$\frac{d\omega}{dt}(t,\theta) = d(\omega(t,\theta)),$$

would not be true, as the left-hand side is trivially zero. Co $V = V_0 + V_1 = 0 + d$, with this new notation we get the *super* different

$$D\omega(t,\theta) = V(\omega(t,\theta$$

which can immediately be splitted into two equations:

$$D_0\omega(t,\theta) = V_0(\omega(t,\theta)), \qquad D_1\omega($$

and is easy to see that in this context, $\omega(t,\theta)$ is a solution to (5.7) as the parameter of the family of integrating automorphisms of a also accept the idea that, when supermanifolds are used, a supert should be used instead of t.

Of course, for certain classes of supervector fields a single commu of $\mathcal{L}_X$ on the supermanifold $\mathcal{M} = (M, \Omega(M))$. In other cases, as anticommuting parameter. By the way, let us remark that a single physics, notably by O. F. Däyi in his study of quantum field theorie dynamical system on a supermanifold as described by a supervecto are a mixture of both cases, that is, of the form (t,θ).

6. The Interpretation of Initial Conditions and the Q

A glance at equations (5.8) tells us that when we have a s homogeneous components of D and V are ω-related, so it follows th

$$[D_0, D_1]\omega(t,\theta) = [V_0, V_1]\omega(t,\theta), \qquad [D_1, D$$

In the classical case, we have only one operator, $D_0 = d/dt$, and o trivially satisfied as $[D_0, D_0] = 0 = [V_0, V_0]$ for any $D_0, V_0 \in \mathcal{X}(M)$ $D_0 = d/dt$ generates the unique one-dimensional (abelian) Lie alg parameters has the structure of a group.

In the setting of supermanifolds, it is natural to expect that
superfields to be integrated because they say that $\{D_0, D_1\}$ must
(this is because we have two generators, one of which (D_0) is ev
that there are 3 non-isomorphic Lie superalgebras with this dimen
a Lie superalgebra, there must be real constants a and b (with $ab =$

$$[D_0, D_1] = aD_1, \qquad [D_1, L$$

and a superdifferential equation $D\omega(t,\theta) = V(\omega(t,\theta))$ will make sens

$$[V_0, V_1] = aV_1, \qquad [V_1, V$$

In contrast with the classical case, these equations are *not* always
example, let W be the supervector field defined by $W_0 = 0$
$\mathcal{M} = (M, \Omega(M))$. Proceeding as we did in Section 5, we get that

$$\omega(t,\theta) = I + \theta\cdot(d +$$

will define a family of automorphisms. However,

$$D_1\omega(t,\theta) = \frac{\partial}{\partial\theta}(I + \theta\cdot(d + i$$

and consequently,

$$W_1(\omega(t,\theta)) = (d + i_X)(I + \theta\cdot(d + i_X)) = d + i$$

so $D_1\omega(t,\theta) \neq W_1\omega(t,\theta)$.

What is wrong with this example? In the example of Section 5 t
graded bracket relations between its homogeneous components w

$$[0,0] = 0, \qquad [0,d] = 0,$$

that is, the homogeneous components of the vector field $V =$
supervector space $\langle\{V_0, V_1\}\rangle$. However, for the case at hand W
components are

$$[0,0] = 0, \qquad [0, d + i_X] = 0, \qquad [d$$

and these do not define a Lie superalgebra structure.

As mentioned, this example seems to imply that not every differe
and indeed, for a long time, it was thought that this is the case. H
[13] were able to construct a procedure to integrate *any* supe
homogeneous components of the supervector field $V = V_0 + V_1$ close
of the conditions on the homogeneous components by introducin
$ev|_{t=t_0}$ (this morphism had already appeared in [32]); to do this,
as

$$ev\,|_{t=t_0} D\omega = ev\,|_{t=t_0}$$

The meaning of this expression, aside of technicalities, is "first,
$t = t_0$." (It is not easy to give, in a few sentences, a motivat
considered as "intuitive". However, we can offer a quick catego
the terminal object is $(\{*\}, \mathbb{R})$, a point with the algebra
$C : (M, \mathcal{A}_M) \to (\{*\}, \mathbb{R})$. On the other hand, every supermanifold h

which induces a graded algebra morphism $f \in \mathcal{A} \mapsto \tilde{f} \in C^{\infty}(M)$
$\delta_p : (\{*\}, \mathbb{R}) \to (M, \mathcal{A}_M)$ simply by declaring that its associated grac
evaluation morphism is determined by these natural morphism:
reconsidering the example of $W = W_1 = d + i_X$, for which we have fou

$$D_1 \omega(t, \theta) = W_1(\omega(t, \theta))$$

Applying the evaluation morphism, the term $\theta \cdot \mathcal{L}_X$ vanishes, and we

$$ev \,|_{t=t_0} D_1 \omega(t, \theta) = ev \,|_{t=t_0}$$

Thus, the introduction of the evaluation morphism allows us to giv
superdifferential equation: the imposition of initial conditions
homogeneous components in such a way that these verify the Lie
be effectively integrated. In this way, the theory in the graded set
of the classical case. We refer the reader to [13] for the details anc

7. Covariant Superderivatives and the Lie Supergrou

In physics, it is common to find expressions involving the
supermechanics, the covariant superderivative is the superfield θ
explore the connection between this superfield and the integrating
no matter which supergroup addition law one uses in the paramet
unique (and always the same) local solution to a given
$\partial / \partial t, \theta(\partial / \partial t) + \partial / \partial \theta$ plays exactly the same role as the pair