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Research Article

The Integrated Density of States for an Multiparticle Homogeneous Model and A to the Anderson Model

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Abstract

For a system of n interacting particles moving in the background c single particle Hamiltonian admits a density of states, so does th integrated density of states coincides with that of the free pa Anderson model, we prove regularity properties of the integrated c

1. Introduction

Recently, models describing interacting quantum particles in a ra 3]). We consider n interacting particles moving in a “homogene space $\mathbb{R}^d$. A typical example of what we mean by a “homogeneo potential. The goal of the present paper is twofold.

First, we prove that if the Hamiltonian of the single particle in t density of states (IDS), then, so does the interacting n-particle Ha prove the claim for the noninteracting n-particle system and noninteracting and interacting system is the same. These two ste Anderson model in $\mathbb{R}^d$.

Note that, in general, knowledge of the integrated density of st counting functions of the finite volume restrictions of the random it is a major tool in the study of the spectrum. Therefore, the sec finite volume normalized counting function which lead to a Wegn developed for the one-particle Hamiltonian.

1.1. The Interacting Multiparticle Model

The noninteracting n-particle Hamiltonian satisfies $H_0^n = -\Delta + V_{ext}^n$ kinetic energy of the n particles. As all the particles are in the sam

$$V_{ext}^n(x_1, \dots, x_n) = \sum_{k=1}^{n}$$

Hence, the noninteracting n-particle Hamiltonian is a sum of or particle potential V^1, we assume that

(H.1.a) $(V^1)_+ := \max\{V^1, 0\}$ is locally square integrable a bounded potential, that is, $\mathcal{D}((V^1)_-) \supseteq \mathcal{D}(-\Delta)$ and for all $\alpha > 0$, th

$$\|(V^1)_-\phi\| \le \alpha\|\Delta\phi\| +$$

(H.1.b) the operator H^1 admits an integrated density of sta restriction of H^1 to a cube $\Lambda(0, L)$ centered at 0 of side-length,

$$N_1(E) := \lim_{L\to+\infty} L^{-d}\,\mathrm{Trace}\,($$

Assumption (H.1.a) implies essential self-adjointness of $-\Delta + V^1$ on

$$V_{ext}^n = (V_{ext}^n)_+ - (V_{ext}^n)_-, \qquad (V_{ext}^n)_\pm(x_1,$$

where

(i) $(V_{ext}^n)_-$ is infinitesimally $-\Delta$-bounded, that is, (1.2) hold $\mathbb{R}^{nd}$;

(ii) $(V_{ext}^n)_+$ is nonnegative locally square integrable.

The self-adjoint extensions of $-\Delta + V^1$ and $-\Delta + V_{ext}^n$ are again der follows.

Classical models for which the IDS is known to exist include perio operators (see, e.g., [5]).

In the definition of the density of states, we could also have co conditions.

The interacting n-particle Hamiltonian is of the form

$$H^n := -\Delta + V_I^n + V_e$$

where

$$V_I^n(x_1, \ldots, x_n) := \sum_{1 \le k < l \le n}$$

is a localized repulsive interaction potential generated by the partic

(H.2) $V : \mathbb{R}^d \to \mathbb{R}$ is measurable nonnegative locally square int

The standard repulsive interaction in three-dimensional space is

some cases, due to screening, it must be replaced by the Yukawa's

Finally, we make one more assumption on both V^1 and V_I; we assur

(H.3) the operator $V_I^n(H_0^n - i)^{-1}$ is bounded.

Assumption (H.3) is satisfied in the case of the Coulomb and Yuk

self-adjoint on $\mathcal{D}(H_0^n) \subseteq \mathcal{D}(-\Delta)$, hence $\|V_I^n(H_0^n - i)^{-1}\| \le \|V_I^n(-\Delta - i)^{-1}$

due to closed graph theorem and $\|V_I^n(-\Delta - i)^{-1}\| < \infty$ for Coulomb

Theorem X.16].

2. The Integrated Density of States

We now compute the IDS for the n-particle model. Let $\Lambda_L = \Lambda(0, L)$

and write $\Lambda_L^n = \Lambda_L \times \cdots \times \Lambda_L$ for the product of n copies of Λ_L. We

Hamiltonian H^n to Λ_L^n with Dirichlet boundary conditions by H_L^n. Cl

H_L^n is bounded from what follows with compact resolvent. Hence,

functions

$$N_L(E) := L^{-nd} \operatorname{Trace}(\mathbf{1}_{]-\infty}$$

As usual, N, the IDS of H^n is defined as the limit of $N_L(E)$ when

states measure applied to a test function φ as the limit of L^-

nonnegative measure. It is a classical result that the existence of

equivalent [5].

2.1. The IDS for the Noninteracting n-Particle System

Recall that, by assumption (H.1.b), the single particle model H^1
measure denoted, respectively, by N_1 and ν_1.

Let $H_{0,L}^n$ be the restriction of H_0^n to Λ_L^n with Dirichlet boundary conc

Lemma 2.1. The IDS for the noninteracting n *-particle Boltzmann r*

$$N_{\mathrm{ni}}(E) := \lim_{L\to\infty} \frac{1}{L^{nd}} \operatorname{Trace}(\mathbf{1}_{]-}$$

exists and satisfies

$$N_{\mathrm{ni}} = N_1 * \nu_1 * \cdots *$$

Let us comment on this result. First, the convolution product in (2
are supported on half-axes of the form $[a,+\infty)$; this results from as
from what follows, one will need some estimate on the decay of
prove it); such estimates are known for some models (see, e.g., [5

Proof. The operator H_0^n is the sum of n commuting Hamiltonians
$H_{0,L}^n$, its restriction to the cube Λ_L^n. As the sum decomposition
eigenvalues of $H_{0,L}^n$ are exactly the sum of n eigenvalues of H^1 rest

$$\operatorname{Trace}(\mathbf{1}_{]-\infty,E]}(H_{0,L}^n)) = (\widetilde{N}_1^L * \widetilde{\ }$$

where $\widetilde{N}_1^L(E)$ is the eigenvalue counting function for H^1 restricted
The normalized counting function and measure, N_1^L and ν_1^L, are def

$$N_1^L = \frac{1}{L^d}\widetilde{N}_1^L, \qquad \nu_1^L =$$

The existence of the density of states of H^1 then exactly says th
The convergence of $N_1^L * \nu_1^L * \cdots * \nu_1^L$ to $N_1 * \nu_1 * \cdots * \nu_1$ is t
bicontinuous operation on distributions. This completes the proof o

Let us now say a word on the boundary conditions chosen to defin
volume limit of the normalized counting for Dirichlet eigenvalu
Hamiltonian has an IDS defined as the infinite-volume limit of the
does the noninteracting n-body Hamiltonian. Moreover, in the ca
Hamiltonian, they also coincide for the noninteracting n-body Har
then sees that the integrated densities of states for both the on
positive mixed boundary conditions also exist and coincide wit
boundary conditions.

2.2. Existence of the IDS for the Interacting n-Particle Syste

Let H_L^n denote the restriction of H^n to the box Λ_L^n with Dirichlet bou

Theorem 2.2. Assume (H.1), (H.2), and (H.3) are satisfied. For any

$$\frac{1}{L^{nd}} \operatorname{Trace}[\varphi(H_L^n) - \varphi(H_{0,L}^n$$

As the density of states measure of H^n is defined by

$$\langle \varphi, dN \rangle = \lim_{L\to+\infty} \frac{1}{L^{nd}} \operatorname{Trace}$$

we immediately get the following corollary.

Corollary 2.3. Assume (H.1), (H.2), and (H.3) are satisfied. The I
H^n exists and coincides with that of the noninteracting model H_0^n; I

$$N = N_{ni} = N_1 * \nu_1 * \cdots$$

Note that, in view of the remark concluding Section 2.1, we see th
n-body Hamiltonian is independent of the boundary conditions if th

In Corollary 2.3, we dealt with the Boltzmann statistic, that is, w
both the Fermi and the Bose statistics, that is, if one restricts t
functions. One defines the following:

(i) for the Fermi statistics, the Fermi integrated density of s

$$\langle \varphi, dN^F \rangle = \lim_{L \to +\infty} \frac{n!}{L^{nd}} \text{Trace}$$

where $\wedge_n L^2(\Lambda_L^1)$ denotes n-fold antisymmetric tensor product o

(ii) for the Bose statistics, the Bose integrated density of sta

$$\langle \varphi, dN^B \rangle = \lim_{L \to +\infty} \frac{n!}{L^{nd}} \text{Trace}$$

where $\overset{s}{\bigoplus}_n L^2(\Lambda_L^1)$ denotes n-fold symmetric tensor product of L^2

Let us now discuss shortly the Bose and Fermi counting functio
Hamiltonian restricted to a finite cube) in the free case (i.e., whe
and let $E_1(L) \le E_2(L) \le \cdots$ be the eigenvalue of the single particle I
three counting functions are then given by

$$\begin{aligned}
\#_L(E) :&= \#\{\text{eigenvalues of } H_{0,L}^n \text{ on } L^2(\\
&= \#\{(j_1, j_2, \ldots, j_n) : E_{j_1}(L) + E_{j_2}(\\
\#_L^F(E) :&= \#\{\text{eigenvalues of } H_{0,L}^n \text{ on } \wedge_n \\
&= \#\{(j_1, j_2, \ldots, j_n) : j_1 < j_2 < \cdots < j \\
\#_L^B(E) :&= \#\{\text{eigenvalues of } H_{0,L}^n \text{ on } \overset{s}{\bigoplus} \\
&= \#\{(j_1, j_2, \ldots, j_n) : j_1 \le j_2 \le \cdots \le j
\end{aligned}$$

Hence,

$$n!\,\#_L^F(E) \le \#_L(E) \le n!$$

Uniformly in L, the eigenvalues $(E_j(L))_{j \ge 1}$ are lower bounded by,
then, for $k = 1, \ldots, n$, one has $E_{j_k}(L) \le E + Cn$ so that $j_k \le \tilde{N}_1^L(E + Cn)$

$$\begin{aligned}
0 &\le \#_L^B(E) - \#_L^F(E) \\
&= \#\left\{(j_1, j_2, \ldots, j_n);\ \begin{array}{l} j_1 \le j_2 \le \cdots \le j_n \\ E_{j_1}(L) + E_{j_2} \end{array}\right. \\
&\le \tilde{C} L^{d(n-1)}.
\end{aligned}$$

Thus, dividing (2.12) and (2.13) by L^{nd} and taking the limit $L \to +\infty$

states are equal to the Boltzmann one. Theorem 2.2 then gives the

Corollary 2.4. Assume (H.1), (H.2), and (H.3) are satisfied. One ha

Proof. We take some $q > nd/2$ and specify the appropriate choice exists $\zeta > 0$ such that

$$-\infty < -\zeta \leq \min\Big(\inf_{L \geq 1}\{\inf[\sigma(H^n_{0,L}) \cup \sigma(H^n_L)$$

Let $y = y(1/2)$ be given by (1.2) for $a = 1/2$. Fix $\lambda_0 > \zeta + 2y + 1$.

By (2.14), we only need to prove (2.6) for $\varphi \in C_0^\infty(\mathbb{R})$ supported in analytic extension of the function $x \mapsto (x+\lambda_0)^q \varphi(x) \in C_0^\infty(\mathbb{R})$, that is

(i) $\tilde{\varphi} \in \mathcal{S}(\{z \in \mathbb{C} : |\Im z| < 1\}$,

(ii) for any $k \in \mathbb{N}$, the family of functions $(x \mapsto (\partial\tilde{\varphi}/\partial\bar{z})(x+iy)$

The functional calculus based on the Helffer-Sjöstrand formula imp

$$\varphi(H^n_L) - \varphi(H^n_{0,L}) = \frac{i}{2\pi}\int_{\mathbb{C}} \frac{\partial\tilde{\varphi}}{\partial\bar{z}}(z)[(H^n_L+\lambda_0)^{-q}(H^n_L - z)^{-}$$

In the following, we apply an idea, which has already been used in resolvent equality, the integrand in (2.15) is written as

$$\begin{aligned}
&(H^n_L+\lambda_0)^{-q}(H^n_L-z)^{-1} - (H^n_{0,L}+\lambda_0\\
&= (H^n_{0,L}+\lambda_0)^{-q}[(H^n_L-z)^{-1} - (\\
&\quad +[(H^n_L+\lambda_0)^{-q} - (H^n_{0,L}+\lambda_0)^{-}\\
&= -(H^n_{0,L}+\lambda_0)^{-q}(H^n_{0,L}-z)^{-1}(V\\
&\quad -\sum_{l=1}^{q}(H^n_{0,L}+\lambda_0)^{l-q-1}(V^n_l)(H
\end{aligned}$$

Estimating the trace of (2.16), we choose $\varepsilon > 0$ and write

$$V^n_i = V^n_i \cdot \mathbf{1}_{\{|V^n_i| \leq \varepsilon\}} + V^n_i$$

and note that $V^n_i \cdot \mathbf{1}_{\{|V^n_i| \leq \varepsilon\}}$ is bounded by $\|V^n_i \cdot \mathbf{1}_{\{|V^n_i| \leq \varepsilon\}}\| \leq \varepsilon$. A

$$\operatorname{supp}(V^n_i \cdot \mathbf{1}_{\{|V^n_i| > \varepsilon\}}) \subseteq \bigcup_{j=1}^{n} \bigcup_{\substack{i=1\\ i \neq j}}^{n} \{(x_1, \ldots, x_n)$$

As, by assumption (H.2), V tends to 0 at infinity, (2.18) implies t that

$$\mu(\{|V^n_i| > \varepsilon\} \cap \Lambda^n_L) \leq C(n$$

where $\mu(\cdot)$ denotes the Lebesgue measure. Using decomposition (2

$$|\mathrm{Trace}\,(H^n_{0,L}+\lambda_0)^{-q}(H^n_{0,L}-z)^{-1}(V^n_i)$$

$$\leq \frac{\varepsilon}{|\Im z|^2}\,\mathrm{Trace}\;|\;(H^n_{0,L}+\lambda_0)^{-q}\;|\;+$$

$$\cdot\,\mathrm{Trace}\;|\;(H^n_{0,L}+\lambda_0)^{-q}\mathbf{1}_{\{|V^n_i|>\varepsilon\}}$$

$$\leq \frac{\varepsilon}{|\Im z|^2}\|(H^n_{0,L}+\lambda_0)^{-1}\|^q_{\mathcal{I}_q}+\frac{1}{|\Im z|^2}\|($$

$$\cdot\,\|(H^n_{0,L}+\lambda_0)^{-1}\mathbf{1}_{\{|V^n_i|>\varepsilon\}\cap\Lambda^n_L}\|$$

where $\|\cdot\|_{\mathcal{I}_q}$ denotes the qth Schatten class norm (see [8]) and

cyclicity of the trace yields

$$|\mathrm{Trace}(H^n_{0,L}+\lambda_0)^{l-q-1}(V^n_i)(H^n_L+\lambda_0)^{-l}(H^n_L-z)^{-1}$$

$$\leq \mathrm{Trace}\;|\;(H^n_L+\lambda_0)^{-l}(H^n_{0,L}+\lambda_0)^{l-q-1}(V^n_i)(H$$

$$\leq \|(H^n_L+\lambda_0)^{-l}(H^n_{0,L}+\lambda_0)^{l}\|\cdot\,\mathrm{Trace}\;|\;(H^n_{0,L}+\lambda$$

$$\leq \frac{C}{|\Im z|}\|(H^n_{0,L}+\lambda_0)^{-1}\|^{q-1}_{\mathcal{I}_q}\cdot\,\|(H^n_{0,L}+\lambda_0)^{-1}\mathbf{1}_{\{|V$$

$$+C\frac{\varepsilon}{|\Im z|}\|(H^n_{0,L}+\lambda_0)^{-1}\|^q_{\mathcal{I}_q}.$$

We are now left with estimating $\|(H^n_{0,L}+\lambda_0)^{-1}\|_{\mathcal{I}_q}$ and $\|(H^n_{0,L}+\lambda_0)^{-1}$

on nd. Therefore, we compute

$$\|(H^n_{0,L}+\lambda_0)^{-1}\mathbf{1}_{\{|V^n_i|>\varepsilon\}\cap\Lambda^n_L}\|_{\mathcal{I}_q}\;\leq\|(H^n_{0,L}+\lambda$$

$$\cdot\,\|(-\Delta_{\Lambda^n_L}+$$

where $-\Delta_{\Lambda^n_L}$ is the Dirichlet Laplacian on Λ^n_L. We use the decom

infinitesimal $-\Delta$-boundedness on $(V^n_{\mathrm{ext}})_-$, [4, Theorem X.18] and the

$$|\langle\phi,(V^n_{\mathrm{ext}})_-\phi\rangle|\leq\frac{1}{2}\langle\phi,-\Delta_{\Lambda}$$

As $\lambda_0>2\gamma+1$, one has

$$H^n_{0,L}+\lambda_0\geq-\Delta_{\Lambda^n_L}+(V^n_{\mathrm{ext}})_-+\lambda_0\geq\frac{1}{2}(-\Delta_{\Lambda^n_L}$$

Thus, the operator $H^n_{0,L}+\lambda_0$ is invertible and

$$(H^n_{0,L}+\lambda_0)^{-1}\leq 2(-\Delta_{\Lambda^n_L}$$

Let $(\mu_{\underline{j}})_{\underline{j}}$ and $(\phi_{\underline{j}})_{\underline{j}}$, respectively, denote the eigenvalues and eigen

$\underline{j}$ runs over $(\mathbb{N}^{nd})^*$). For $q\in\mathbb{N}$ such that $2q>nd$, we compute

$$\|(H^n_{0,L}+\lambda_0)^{-1}(-\Delta_{\Lambda^n_L}+\lambda_0)^{1/2}\|^{2q}_{\mathcal{I}_{2q}} = \sum_{\underline{j}\in\mathbb{N}^{nd}}(\mu_{\underline{j}}(-\Delta_{\Lambda}$$

$$\leq 2^{2q}\sum_{\underline{j}\in\mathbb{N}^{nd}}(\mu_{\underline{j}}($$

$$= 2^{2q}\sum_{\underline{j}\in\mathbb{N}^{nd}}(\mu_{\underline{j}}($$

The last estimate is a direct computation using the explicit form of

By [6, Lemma 2.2], we know that, for $q\in\mathbb{N}$ such that $2q>nd$, subset $\Lambda'\subseteq\Lambda^n_L$, one has

$$\|(-\Delta_{\Lambda^n_L}+\lambda_0)^{-1/2}\mathbf{1}_{\Lambda'}\|^{2q}_{\mathcal{I}_{2q}}\leq$$

Choosing $\Lambda'=\{|V^n_i|>\varepsilon\}\cap\Lambda^n_L$ and taking (2.19) into account, th that there exists c, depending only on q (and the bound in assumpt

$$\text{Trace}\,|\,(H^n_L+\lambda_0)^{-q}(H^n_L-z)^{-1}-(H^n_{0,L}+$$

$$\leq c\Big(\frac{\varepsilon}{|\Im z|^2}L^{nd}+\frac{1}{|\Im z|^2}L^{nd-(d/2q)}+\frac{\varepsilon}{|\Im}$$

By using this inequality in (2.15), we get (2.6) as $\tilde{\Phi}$ being almo approaches the real line. Thus, we completed the proof of Theorem

3. Application to the Interacting Multiparticle Ande

In the interacting multiparticle Anderson model, we consider a ran particle Anderson potential is of the form

$$V^1(\omega,x)=\sum_{j\in\mathbb{Z}^d}\omega_j u$$

with a family $\omega_j:\Omega\to\mathbb{R}$ of random variables on $(\Omega,\mathbf{P})$. This one- “background” potential

$$V^n(\omega,x_1,\ldots,x_n)=\sum_{k=1}^{n}V$$

and the interacting n-particle Hamiltonian reads as

$$H^n(\omega)=-\Delta+V^n_i+V$$

For the Anderson model, it is known under rather general assu counting function defined in assumption (H.1.b) converges al nondecreasing function of E. Its discontinuity set is countable. By | the normalized counting function defined in assumption (H.1.b) th now apply the results of the last section and get a $\mathbf{P}$-almost sure noninteracting and interacting n-particle system. Note that $(j,j,\ldots,j)\in\mathbb{Z}^{nd}$ leave $H^n(\omega)$ invariant. Hence, for an application of the proof of existence and $\mathbf{P}$-almost sure constancy of N, there are

One of the interesting properties of the integrated density of st
important role in the theory of localization for random one-partic
play through a Wegner estimate, that is, an estimate of the type

$$\mathbb{E}(\text{Trace } \mathbf{1}_{]E_0,E_0+\eta]}(H^n_\Lambda))$$

On the other hand, Corollary 2.3 directly relates the regularity of t
of the single particle Hamiltonian. The regularity of the IDS of t
interest recently (see, e.g., [11, 12]).

We now prove a Wegner estimate; for convenience, we assume the

(H.A.2) The single-site potential u is nonnegative, compactly
such that $u(x) \geq c$ for $x \in [-(1/2), 1/2]^d$.

For the proof of a Wegner estimate in the interacting n-particle
probabilistic hypothesis like in [13]:

(H.A.3) $(\omega_j : \Omega \to \mathbb{R})_{j \in \mathbb{Z}^d}$ is a family of bounded random varia

When μ_j denotes the conditional probability measure for ω_j at
variables $(\omega_i)_{i \neq j}$, that is, for all $A \in \mathcal{B}(\mathbb{R})$,

$$\mu_j(A) = \mathbf{P}(\{\omega_j \in A \mid (\omega$$

then, a Wegner estimate à la [13] uses the quantity

$$s(\eta) := \sup_{j \in \mathbb{Z}^d} \mathbb{E}\{\sup_{E \in \mathbb{R}} \mu_j([$$

and is stated as follows.

Theorem 3.1. Let us assume (H.A.2) and (H.A.3), and let $\Lambda \subseteq \mathbb{R}$
$H^n_\Lambda(\omega)$ be the restriction of $H^n(\omega)$ to Λ with Dirichlet boundary conc

$$C_W : \mathbb{R} \to [0, \infty[$$
$$E_0 \mapsto C_W(E_0),$$

such that for all $\eta > 0$

$$\mathbb{E}(\text{Trace } \mathbf{1}_{]E_0,E_0+\eta]}(H^n_\Lambda)) \leq C_W$$

In order to prove Theorem 3.1, we prove two preparatory lemmas.

Lemma 3.2. Let $\Lambda \subseteq \mathbb{R}^{nd}$ *be an open bounded cube, then the res*
Dirichlet or Neumann boundary conditions define self-adjoint opera

Proof. V^n_i is infinitesimally $-\Delta$ form bounded according to [4, Theore

$$|\langle \Psi, V^n_i \Psi \rangle| \leq \varepsilon \|\nabla \Psi\|^2 +$$

is true for $\Psi \in H^1(\mathbb{R}^{nd})$, in particular (3.9) is true for $\Psi \in \mathcal{D}(-\Delta_\Lambda) =$

representation theorem a self-adjoint operator $H^n_{l,\Lambda} = -\Delta_\Lambda + V^n_l|_\Lambda$.

minimax principle and (3.9), we see that $H^n_{l,\Lambda}$ has compact resolve

$$\langle \Psi, V^n_l \Psi\rangle \le \varepsilon \|\nabla \Psi\|^2 + c_\varepsilon \|\Psi\|^2.$$

uses the extension operator $E_{\Lambda'} : H^1(\Lambda) \to H^1_0(\Lambda')$ to $\Lambda' := \{x \in \mathbb{R}^{nd}$:

$\|E_{\Lambda'}\Psi\|_{H^1} \le c_1\|\Psi\|_{H^1}$ and $\|E_{\Lambda'}\Psi\|_{L^2} \le c_2\|\Psi\|_{L^2}$; see [14, Satz 5.6 and F

(3.9), hence by $V^n_l \ge 0$ and the above properties of $E_{\Lambda'}$ we get for Ψ

$$0 \le \langle \Psi, V^n_l \Psi\rangle \le \langle E_{\Lambda'}\Psi, V^n_l E_{\Lambda'}\Psi\rangle \le \varepsilon \|\nabla (E_{\Lambda'}\Psi)$$

$$= \varepsilon \|(E_{\Lambda'}\Psi)\|^2_{H^1} + (b_\varepsilon - \varepsilon)\|E_{\Lambda'}\Psi\|^2_{L^2} \le \varepsilon c_1^2 \|\nabla$$

which is (3.10). With (3.10) at hand, the proof for Neumann bounc

Lemma 3.3. Let one assumes (H.A.2) and (H.A.3), and let $\Lambda \subseteq \mathbb{R}^n$

$\Lambda_{\mathbf{j}} := \Lambda \cap \Lambda(\mathbf{j}, 1) \ne \varnothing$ *(here,* $\Lambda(\mathbf{j}, 1) = \{ | x - j_k | \le 1/2,\ 1 \le k \le n\}$ *), the*

$$\mathbb{E}\{\langle f, \mathbf{1}_{]E_0, E_0+\eta]}(H^n_\Lambda) f\rangle\} \le$$

Proof. For every $j \in \mathbb{Z}^d$, we define $u_j : \mathbb{R}^{nd} \to \mathbb{R}$ by

$$u_j(x_1, \ldots, x_n) := \sum_{k=1}^{n} u$$

and set $\tilde{\omega}_j = (\omega_l)_{l \ne j}$. Fix a component of $\mathbf{j}$, say j_1, then we get a de

$$V^n(\omega, x_1, \ldots, x_n) = \omega_{j_1} u_{j_1}(x_1, \ldots, x_n)$$

of the random potential $V^n(\omega)$, and the same is true for $H^n_\Lambda(\omega)$:

$$H^n_\Lambda(\omega) = -\Delta_\Lambda + \sum_{\substack{l \in \mathbb{Z}^d \\ l \ne j_1}} \omega_l u_l \mathbf{1}_\Lambda + \omega_{j_1} u_{j_1} \mathbf{1}_\Lambda$$

By the covering condition $u\, \mathbf{1}_{[-1/2, 1/2]^d} \ge c$ on the single site-pc

$f = g u_{j_1}$, where $g(x) = f(x) / u_{j_1}(x)$ almost everywhere, so $\|g\| \le c^{-1}\|$

$$\int_{E_0}^{E_0+\eta} dE \langle \varphi, \Im (H - E - i\eta)^{-1} \varphi\rangle \ge \frac{\eta}{4}\langle$$

for every self-adjoint H, see [13], (3.9). The equalities and estimat

into a form, where the results of spectral averaging, [11, Section 3

$$\begin{aligned} \mathbb{E}\langle f, \mathbf{1}_{]E_0, E_0+\eta]}(H^n_\Lambda) f\rangle &= \mathbb{E}\int_{\mathbb{R}} d\mu_{j_1}(\omega_{j_1}) \langle g, u_{j_1} \mathbf{1}_{]E_0, E_0} \\ &\le \frac{4}{\eta} \mathbb{E}\int_{\mathbb{R}} d\mu_{j_1}(\omega_{j_1}) \int_{E_0}^{E_0+\eta} dE\, \Im \langle \\ &\le \frac{8}{c^2}\|f\|^2 s(\eta). \end{aligned}$$

Proof. By (H.A.2) and (H.A.3), we get a $\mathbf{P}$-almost sure bound restrictions $H^n_\Lambda(\omega)$ and $H^n_{\Lambda,N}(\omega)$ of $H^n(\omega)$ to a bounded open cu

define self-adjoint operators with compact resolvent $\mathbf{P}$-almost sur $\Lambda_{\mathbf{j}} := \Lambda(\mathbf{j},1) \cap \Lambda$. Then $\Lambda' := \Lambda \setminus \cup_{\mathbf{j}\in J}\Lambda_{\mathbf{j}}$ has Lebesgue measure 0, s

$$-\Delta_\Lambda \geq -\Delta_{\Lambda,N} \geq -\Delta_{\Lambda\setminus\Lambda',N} = {}_{\mathbf{j}}$$

So with $H^n_{l,\Lambda_{\mathbf{j}},N}$ defined in Lemma 3.2, we get $\mathbf{P}$-almost sure:

$$H^n_\Lambda(\omega) \geq H^n_{\Lambda,N} := \bigoplus_{\mathbf{j}\in J} H^n_{l,\Lambda}$$

By spectral calculus,

$$\text{Trace}\,(\mathbf{1}_{]E_0,E_0+\eta]}(H^n_\Lambda(\omega))) \leq e^{E_0+\eta}\,\text{Trace}\,($$

Let $(\varphi_k(\omega))_{k\in\mathbb{N}}$ be the orthogonal basis of $L^2(\Lambda)$ consisting out of $M(\omega) := \{k \in \mathbb{N} : \mu_k(\omega) \in]E_0, E_0+\eta]\}$, then

$$\begin{aligned}\text{Trace}\,(\mathbf{1}_{]E_0,E_0+\eta]}(H^n_\Lambda(\omega))e^{-H^n_\Lambda(\omega)}) &= \sum_{k\in M(}\\ &\leq \sum_{k\in M(}\\ &\leq \sum_{k\in M(}\\ &= \text{Trace}\end{aligned}$$

where the last estimate follows from Jensen's inequality. Let $(\phi_{k,\mathbf{j}})$ eigenvectors of $H^n_{l,\Lambda_{\mathbf{j}},N}$ to the eigenvalues $E_{k,\mathbf{j}}$, then

$$\text{Trace}(\mathbf{1}_{]E_0,E_0+\eta]}(H^n_\Lambda(\omega))e^{-H^n_{\Lambda,N}}) = \sum_{k\in\mathbb{N}}\sum_{\mathbf{j}\in J}\langle\phi_{k,\mathbf{j}},$$

As $\phi_{k,\mathbf{j}} \in L^2(\Lambda_{\mathbf{j}})$ and $\|\phi_{k,\mathbf{j}}\| \leq 1$, Lemma 3.3 implies

$$\mathbb{E}\langle\phi_{k,\mathbf{j}}, \mathbf{1}_{]E_0,E_0+\eta]}(H^n_\Lambda(\omega))\phi_k$$

As V^n_l is nonnegative, the eigenvalues $E_{k,\mathbf{j}}$ of $H^n_{l,\Lambda_{\mathbf{j}},N} = -\Delta_{\Lambda_{\mathbf{j}},N}$ eigenvalues of $-\Delta_{\Lambda_{\mathbf{j}},N}$. These are known explicitly, see [15, page 2

$$\sum_{k\in\mathbb{N}}\sum_{\mathbf{j}\in J} e^{-E_{k,\mathbf{j}}} \leq \text{Card}\,(J)($$

If the side-length of Λ is bigger than 1, then $\text{Card}(J) \leq 3^{nd}|\Lambda|$, s inequalities (3.20) to (3.24), it implies

$$\mathbb{E}(\text{Trace}\,\mathbf{1}_{]E_0,E_0+\eta]}(H^n_\Lambda)) \leq e^{E_0+\eta+\|V^n\|}$$

Under the assumptions (H.A.2) and (H.A.3), we have

$$N(E) = \mathbb{E}(N(E,\cdot)\mathbf{1}_{\Omega'}) = \mathbb{E}$$

hence by the Wegner estimate we can deduce regularity proper $(\mu_j)_{j\in\mathbb{Z}^d}$ via

$$0 \le N(E+\eta) - N(E) \le C_W(\ell$$

References

1. W. Kirsch, "A Wegner estimate for multi-particle random Ha *Analiza, Geometrii*, vol. 4, no. 1, pp. 121–127, 2008.
2. V. Chulaevsky, "Wegner-Stollmann type estimates for some *Mathematical Physics*, vol. 447 of *Contemporary Mathematic* Providence, RI, USA, 2007.
3. M. Aizenman and S. Warzel, "Localization bounds for multip http://arxiv.org/abs/0809.3436.
4. M. Reed and B. Simon, *Methods of Modern Mathematical Phy* Academic Press, New York, NY USA, 1975.
5. L. Pastur and A. Figotin, *Spectra of Random and Almost-Per* *Mathematischen Wissenschaften*, Springer, Berlin, Germany
6. F. Klopp and L. Pastur, "Lifshitz tails for random Schröding potential," *Communications in Mathematical Physics*, vol. 2
7. F. Klopp, "Internal Lifshits tails for random perturbations of *Mathematical Journal*, vol. 98, no. 2, pp. 335–396, 1999.
8. B. Simon, *Trace Ideals and Their Applications*, vol. 120 of *M* Mathematical Society, Providence, RI, USA, 2nd edition, 20
9. R. Carmona and J. Lacroix, *Spectral Theory of Random Schr* Birkhäuser, Boston, Mass, USA, 1990.
10. P. Stollmann, *Caught by Disorder: Bound States in Random* Birkhäuser, Boston, Mass, USA, 2001.
11. J.-M. Combes, P. D. Hislop, and F. Klopp, "Local and global *Advances in Differential Equations and Mathematical Physics* *Contemporary Mathematics*, pp. 61–74, American Mathema
12. G. Stolz, "Strategies in localization proofs for one-dimensio *of the Indian Academy of Sciences. Mathematical Sciences*,
13. J.-M. Combes, P. D. Hislop, and F. Klopp, "An optimal Wegn continuity of the integrated density of states for random Sch vol. 140, no. 3, pp. 469–498, 2007.
14. J. Wloka, *Partielle Differentialgleichungen*, B. G. Teubner, St
15. M. Reed and B. Simon, *Methods of Modern Mathematical Phy* New York, NY, USA, 1978.