Mathematical Physics

On Lagrangian and Hamiltonian systems with homogeneous trajectories

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(Submitted on 7 Mar 2010)

Motivated by various results on homogeneous geodesics of Riemannian spaces, we study homogeneous trajectories, i.e. trajectories which are orbits of a one-parameter symmetry group, of Lagrangian and Hamiltonian systems. We present criteria under which an orbit of a one-parameter subgroup of a symmetry group G is a solution of the Euler-Lagrange or Hamiltonian equations. In particular, we generalize the `geodesic lemma' known in Riemannian geometry to Lagrangian and Hamiltonian systems. We present results on the existence of homogeneous trajectories of Lagrangian systems. We study Hamiltonian and Lagrangian g.o. spaces, i.e. homogeneous spaces G/H with G-invariant Lagrangian or Hamiltonian functions on which every solution of the equations of motion is homogeneous. We show that the Hamiltonian g.o. spaces are related to the functions that are invariant under the coadjoint action of G. Riemannian g.o. spaces thus correspond to special Ad*(G)-invariant functions. An Ad*(G)-invariant function that is related to a g.o. space also serves as a potential for the mapping called `geodesic graph'. As illustration we discuss the Riemannian g.o. metrics on SU(3)/SU(2).

Comments:	20 pages, LaTeX
Subjects:	**Mathematical Physics (math-ph)**; Differential Geometry (math.DG); Exactly Solvable and Integrable Systems (nlin.SI)
Cite as:	**arXiv:1003.1495v1 [math-ph]**

Submission history

From: Gábor Zsolt Tóth [view email]
[v1] Sun, 7 Mar 2010 17:17:33 GMT (16kb)

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