

Mathematical Physics

Spectral Singularities, Biorthonormal Systems, and a Two-Parameter Family of Complex Point Interactions

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A curious feature of complex scattering potentials $v(x)$ is the appearance of spectral singularities. We offer a quantitative description of spectral singularities that identifies them with an obstruction to the existence of a complete biorthonormal system consisting of the eigenfunctions of the Hamiltonian operator, i.e., $-\frac{d^2}{dx^2} + v(x)$, and its adjoint. We establish the equivalence of this description with the mathematicians' definition of spectral singularities for the potential $v(x) = z_- \delta(x+a) + z_+ \delta(x-a)$, where z_{\pm} and a are respectively complex and real parameters and $\delta(x)$ is the Dirac delta-function. We offer a thorough analysis of the spectral properties of this potential and determine the regions in the space of the coupling constants z_{\pm} where it admits bound states and spectral singularities. In particular, we find an explicit bound on the size of certain regions in which the Hamiltonian is quasi-Hermitian and examine the consequences of imposing PT-symmetry.

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