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On Quantum Mechanics on Noncommutative Quantum Phase Space

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Abstract: In this work, we develop a general framework in which Noncommutative Quantum Mechanics (NCQM), characterized by a space noncommutativity matrix parameter $\theta=\varepsilon_{ij}{}^{k}\,\theta_{k}$ and a momentum noncommutativity matrix parameter $\beta=\varepsilon_{ij}{}^{k}\,\beta_{k}$, is shown to be equivalent to Quantum Mechanics (QM) on a suitable transformed Quantum Phase Space (QPS). Imposing some constraints on this particular transformation, we firstly find that the product of the two parameters θ and β possesses a lower bound in direct relation with Heisenberg incertitude relations, and secondly that the two parameters are equivalent but with opposite sign, up to a dimension factor depending on the physical system under study. This means that noncommutativity is represented by a unique parameter which may play the role of a fundamental constant characterizing the whole NCQPS. Within our framework, we treat some physical systems on NCQPS : free particle, harmonic oscillator, system of two-charged particles, Hydrogen atom. Among the obtained results, we discover a new phenomenon which consists of a free particle on NCQPS viewed as equivalent to a harmonic oscillator with Larmor frequency depending on β, representing the same particle in presence of a magnetic field $\vec{B}=q^{-1}\vec{\beta}$. For the other examples, additional correction terms depending on β appear in the expression of the energy spectrum. Finally, in the two-particle system case, we emphasize the fact that for two opposite charges noncommutativity is effectively feeled with opposite sign.

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