Quantum Physics

Analysis of Lyapunov Method for Control of Quantum States

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The natural trajectory tracking problem is studied for generic quantum states represented by density operators. A control design based on the Hilbert-Schmidt distance as a Lyapunov function is considered. The control dynamics is redefined on an extended space where the LaSalle invariance principle can be correctly applied even for non-stationary target states. LaSalle's invariance principle is used to derive a general characterization of the invariant set, which is shown to always contain the critical points of the Lyapunov function. Critical point analysis of the latter is used to show that, for generic states, it is a Morse function with $n!$ isolated critical points, including one global minimum, one global maximum and $n!-2$ saddles. It is also shown, however, that the actual dynamics of the system is not a gradient flow, and therefore a full eigenvalue analysis of the linearized dynamics about the critical points of the dynamical system is necessary to ascertain stability of the critical points. This analysis shows that a generic target state is locally asymptotically stable if the linearized system is controllable and the invariant set is regular, and in fact convergence to the target state (trajectory) in this case is almost global in that the stable manifolds of all other critical points form a subset of measure zero of the state space. On the other hand, if either of these sufficient conditions is not satisfied, the target state ceases to be asymptotically stable, a center manifold emerges around the target state, and the control design ceases to be effective.

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