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Quaternionic Roots of E 8 Related Coxeter Graphs and Quasicrystals

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Abstract: The lattice matching of two sets of quaternionic roots of F_4 leads to quaternionic roots of E_8 which has a decomposition $H_4 + \sigma H_4$ where the Coxeter graph H_4 is represented by the 120 quaternionic elements of the binary icosahedral group. The 30 pure imaginary quaternions constitute the roots of H_3 which has a natural extension to $H_3 + \sigma H_3$ describing the root system of the Lie algebra D_6. It is noted that there exist three lattices in 6-dimensions whose point group $W(D_6)$ admits the icosahedral symmetry H_3 as a subgroup, the roots of which describe the mid-points of the edges of an icosahedron. A natural extension of the Coxeter group H_2 of order 10 is the Weyl group $W(A_4)$ where $H_2 + \sigma H_2$ constitute the root system of the Lie algebra A_4. The relevance of these systems to quasicrystals are discussed.