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Repulsive and Restoring Casimir Forces Based on Magneto-Optical Effect *

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The Casimir force direction tuned by the external magnetic field due to the magneto-optical Voigt effect is investigated. The magneto-optical effect gives rise to the modified frequency-dependent electric permittivity and thus the electromagnetic properties of the materials can be adjusted to satisfy the condition of the formation of repulsive Casimir force. It is found that between the ordinary dielectric slab and magneto-optical material slab, a repulsive force may exist by adjusting the applied magnetic field. The restoring Casimir force can also be obtained if suitable parameter values are taken. For realistic materials, the repulsive and the restoring force is shown to possibly take place at typical distances in microelectromechanical systems.

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The change of the zero point energy of quantized electromagnetic field in the presence of the boundary surface gives rise to forces on macroscopic bodies. It was first shown theoretically by Casimir in 1948 that a pair of neutral, perfectly conducting parallel plates located in the vacuum attract each other.[1] The existence of the Casimir forces has attracted considerable attention over decades.[2] Various calculation techniques have been developed. Generally the two main calculation methods are the surface mode summation method,[3] and the stress tensor method.[4,5] The Casimir effects for various practical materials[6−8] were investigated, which resulted in various characteristics that can be used to control the force in the possible applications.

In recent years, the Casimir force can be measured in experiment with the development of microelectromechanical and nanoelectromechanical systems (MEMSs) and nanotechnology.[2,3,9] The attractive Casimir forces could lead to stiction problem in MEMS and NEMS. Therefore, the repulsive Casimir force attracts much attention due to this practical significance.[10,11] The study to obtain the repulsive force with different materials, such as the metamaterials, is made in the recent literature.[12−15] Perfect lens made of left-handed metamaterial is introduced in the planar geometry to obtain repulsive Casimir force.[12] The repulsive Casimir force between two parallel metamaterial plates can occur under certain conditions.[13−15] We have focused in the previous work[15] on the stable equilibrium at the force direction transition. The permittivity of some semiconductors can be changed by an applied magnetic field due to the magneto-optical effect.[16] In this Letter, we investigate the possibility of obtaining the repulsive and restoring Casimir force by changing the external magnetic field, focusing on the effect under the influence of the frequency-dependent electromagnetic properties of the materials and its relation with the distance between the materials.

We consider the structure made of a pair of parallel infinite slabs, A and B, of the same thickness d and separation a in free space. Based on the stress tensor method using the properties of the macroscopic field operators,[4,6] here we calculate the Casimir force with a convenient Green function method. The problem is treated by evaluating the vacuum radiation pressure on the slabs, which is determined by the zz component of the stress tensor T_{zz}. With β being the z component of the wave vector, given by $\beta(\omega,k)=\sqrt{\omega^2/c^2-k^2}$ and k being the component of the wave vector parallel to the slab surface, the Casimir force, i.e., T_{zz} in the layer separated the slabs, can be expressed after adopting the form of scattering Green function as given in Ref. [17] as

$$F_C=T_{zz}=-\frac{\hbar}{2\pi^2}\mathrm{Re}\int_0^\infty d\omega\int_0^\infty k\beta dk \times\sum_q D_q^{-1}r_q^A r_q^B e^{2i\beta a}, \qquad (1)$$

where $q=s,p$ stands for different polarized plane wave, $D_q=1-r_q^A r_q^B e^{2i\beta a}$ is the term for multiple reflection between two surfaces, $r_q^{A(B)}$ is the reflection coefficients of the left (right) slab A (B) for q-polarized wave, and can be calculated by transfer matrix method. We have previously discussed that in general, the repulsive Casimir effect is to be expected when the two parallel slabs have different electromagnetic properties:[15] one can obtain the repulsive force if one of the slabs is mainly electric (indicated as $|\mathrm{Re}[\epsilon_{A(B)}]/\mathrm{Re}[\mu_{A(B)}]|>1$) while the other mainly magnetic (indicated as $|\mathrm{Re}[\epsilon_{A(B)}]/\mathrm{Re}[\mu_{A(B)}]|<1$). Furthermore, the force direction transition from repulsion to attraction for the increasing slab separation that may be useful in practice, called the restor-

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ing Casimir force, can also be observed owing to the difference of the frequency-dependent electromagnetic properties between two slabs.

In the following we consider the Casimir effect between the ordinary dielectric material (slab A) with $\epsilon_A = 2$, $\mu_A = 1$ and the magneto-optical material (slab B). The magneto-optical effect gives rise to the possibility of adjusting the material's permittivity by means of an external magnetic field. In the famous magneto-optical Voigt effect, the permittivity for the electric field of electromagnetic waves parallel to the external magnetic field keeps unchanged, while for the electric field of electromagnetic waves perpendicular to the external magnetic field, the permittivity will be modified with the presence of the external magnetic field and is given by[18]

$$\epsilon(\omega) = \epsilon_0 \left(1 - \frac{\omega_p^2 \left(\omega^2 + i\omega/\tau\right) - \omega_p^4}{\left(\omega^2 + i\omega/\tau\right)\left(\omega^2 - \omega_c^2 - \omega_p^2 + i\omega/\tau\right)}\right), \tag{2}$$

where ϵ_0 is static dielectric constant, ω_p is the screened plasma frequency and τ is the relaxation time that leads to a damping term when the absorption is concerned. $\omega_c = eB/m^*c$ is the cyclotron frequency, which varies linearly with the external magnetic field B; m^* is the effective mass and c is the speed of light in vacuum.

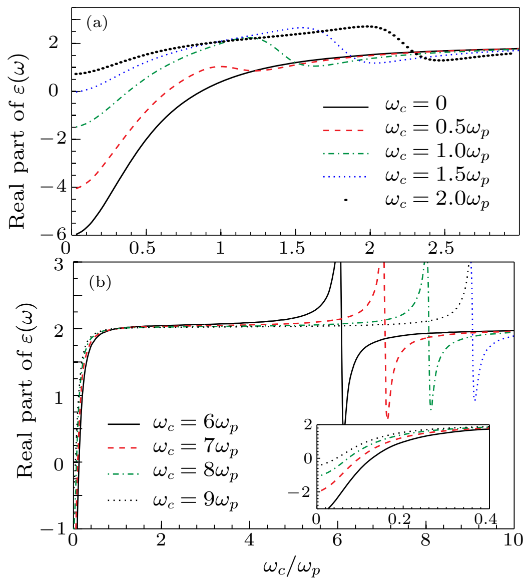

Fig. 1. The real part of electric permittivity $\epsilon(\omega)$ of the magneto-optical material for different cyclotron frequencies. The static dielectric constant $\epsilon_0 = 2$, the absorption is taken to be $1/\tau = 0.5\omega_p$ (a) and $1/\tau = 0.1\omega_p$ (b).

The frequency dependence of the real part of the permittivity $\epsilon(\omega)$ for external different magnetic fields and different τ are shown in Fig. 1. The magnetic field influences the resonance location of the permittivity due to the cyclotron-plasmon hybridization, the resonance frequency becomes higher with the increasing ω_c. From the above, one can find that for two slabs with trivial permeability $\mu = 1$, the different permittivities, one larger and the other smaller than unity, may result in the repulsion between them. Therefore, we will remark the frequency region where the magnitude of the permittivity is no larger than unity. Especially at low frequencies the $|\mathrm{Re}[\epsilon(\omega)]| < 1$ regions may offer main contribution to the repulsive force for small slab separation (to be discussed later on). For relatively high damping [Fig. 1(a)], the magnitude of the resonance caused by the external magnetic field is small and $|\mathrm{Re}[\epsilon(\omega)]| < 1$ region can be increased by adjusting the magnetic field. This region at low frequencies may also be remarkably widened by adjusting the magnetic field for low damping [Fig. 1(b)], but the applied magnetic field needs to be sufficiently strong.

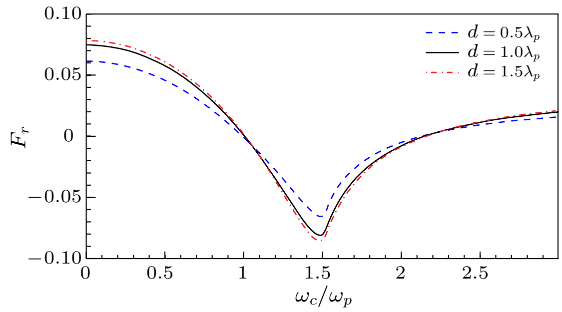

Fig. 2. The relative Casimir force F_r between an ordinary dielectric slab and a magneto-optical material slab as a function of external magnetic field for different slab thicknesses. The slab separation $a = \lambda_p$ and for the magneto-optical material, $1/\tau = 0.5\omega_p$, $\epsilon_0 = \epsilon_A = 2$.

We proceed now to the study of formation of the repulsive and restoring Casimir force with the presence of the external magnetic field. In the following, the relative Casimir force $F_r = F_C/F_0$ is used in discussion, where $F_0 = \hbar c\pi^2/240a^4$ is the well-known formula for the Casimir force between two perfectly conducting plates and the slab separation and the thickness are measured in units $\lambda_p = c/\omega_p$. In Fig. 2, we show the dependence of the force on external magnetic field for a fixed slab separation. There is Casimir attraction ($F_r > 0$) between the slabs when the external magnetic field is weak. The attractive force decreases with the increasing ω_c and becomes repulsive ($F_r < 0$) for certain value of ω_c. The repulsive force grows larger and attains a maximum and then decreases and eventually returns to attraction again. The increase of the slab thickness enhances the magnitude of the force.

It can be seen that the above result is generally consistent with the analysis of the electromagnetic properties of the slabs. The discrepancy between the permittivities of two slabs determines whether there is repulsive force. Slab A is the dielectric material with permittivity greater than unity, thus the contribution to the formation of repulsion comes from the frequency band where the magnitude of ϵ of slab B, the magneto-optical material, is smaller than unity. For small values of ω_c, the region of $|\mathrm{Re}[\epsilon(\omega)]| < 1$ is relatively narrow, as Fig. 1(a) indicates. Hence the entire integral of the Casimir force calculation leads to attractive force. By enhancing the external magnetic field, the $|\mathrm{Re}[\epsilon(\omega)]| < 1$ region can be widened to low frequencies and more contribution to the repulsion emerges, which lowers the attractive force down to zero and even to repulsive force. With further increasing ω_c, however, the $|\mathrm{Re}[\epsilon(\omega)]| < 1$ region may stop growing and start to shrink, then the repulsive force reaches its maximum and decreases and returns to attraction.

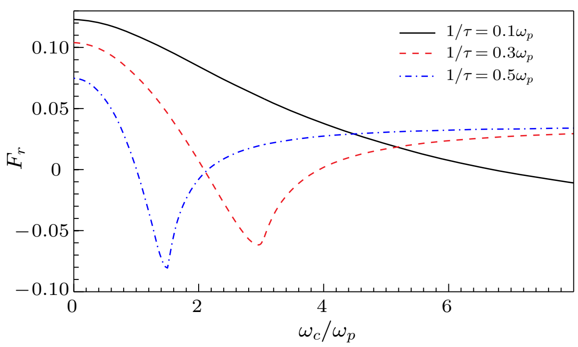

Fig. 3. The relative Casimir force F_r between an ordinary dielectric slab and a magneto-optical material slab as a function of external magnetic field for different absorptions of the magneto-optical material with $\epsilon_0 = \epsilon_A = 2$. The slab separation $a = \lambda_p$ and the slab thickness $d = \lambda_p$.

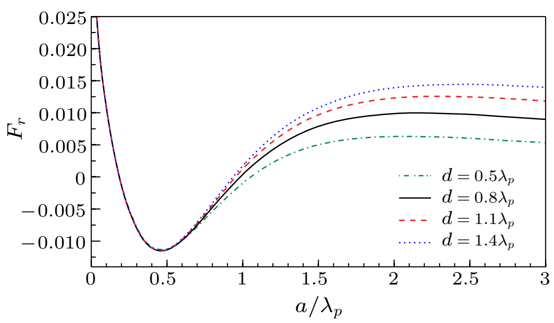

Fig. 4. The relative Casimir force F_r between an ordinary dielectric slab and a magneto-optical material slab as a function of the slab separation for different slab thicknesses. For the magneto-optical material, $1/\tau = 0.5\omega_p$, $\epsilon_0 = \epsilon_A = 2$ and the cyclotron frequency $\omega_c = \omega_p$.

Different absorption of the magneto-optical material may also affect the Casimir effect as shown in Fig. 3. As stated above, for the relatively large relaxation time τ, the stronger magnetic field is necessary to obtain wider $|\mathrm{Re}[\epsilon(\omega)]| < 1$ region. Therefore, with enlarged relaxation time, i.e., the reduced absorption, the region where the repulsive Casimir force can be obtained may move to greater values of ω_c.

We then study the attraction-repulsion transition when the force varies with the changing slab separation. In Fig. 4 we present the dependance of Casmir force on slab separation for different slab thicknesses under fixed magnetic field. One can adjust the external magnetic field to satisfy the condition of formation of repulsive force stated above at certain slab separation. Generally, the force is attractive at short distances and as the slabs get further away from each other, the force is lowered and may become repulsive at certain distance. Furthermore, it begins to grow larger and reaches a maximum and then decreases to zero where there is an equilibrium, and eventually turns to be attractive. This latter direction transition forms a restoring force, which can be used to stabilize the system, since it may maintain the slab separation at the vicinity of the equilibrium.

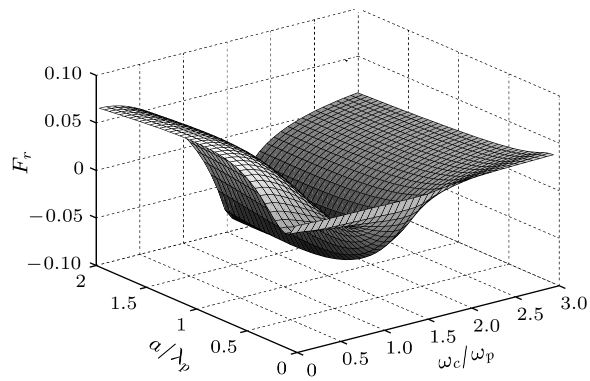

Fig. 5. The relative Casimir force F_r between an ordinary dielectric slab and a magneto-optical material slab as a function of the slab separation and the external magnetic field. For the magneto-optical material, $1/\tau = 0.5\omega_p$, $\epsilon_0 = \epsilon_A = 2$ and the slab thickness $d = \lambda_p$.

At short distances, the corresponding effective frequency range that has the main contribution to the Casimir force integral is rather wide. As seen in Fig. 1, magnitude values of ϵ larger than unity dominates over the whole frequency band, thus there is attraction at short distances. The effective frequency range is narrowed with the increasing slab separation and when the range mainly consists of the frequencies where magnitudes of $\epsilon(\omega)$ are no larger than unity, the force becomes repulsive, which is the first direction transition in Fig. 4. With the further increasing distance, the main contribution to the force comes from rather low frequencies, so it changes to attraction due to $|\mathrm{Re}[\epsilon(\omega)]| > 1$ at low frequencies.

With the variation of the external magnetic field, the $|\mathrm{Re}[\epsilon(\omega)]| < 1$ region can extend down to zero frequency [e.g., the frequency dependence of $\epsilon(\omega)$ for $\omega_c = 1.5\omega_p$ in Fig. 1(a)], which does not lead to the expected restoring force, the force will keeps repulsive when the slab separation increases. Figure 5 shows the dependence of the force on the slab separation while the external magnetic field varies. When the magnetic field is very weak, there are always attractive forces, since the contribution to repulsive force from the effective frequency band corresponding to any distance may almost be neglected. With the enhanced magnetic field, the repulsive forces begin to exist within a certain distance region. The repulsion region fades out and the attraction region fades in with the increasing distance, which gives rise to the restoring force. When ω_c keeps on increasing, the repulsion region is expanded. With the increasing distance the repulsive force may no longer return to attraction, where the restoring force disappears. The distance where the attraction becomes repulsion moves to larger values with the increasing ω_c greater than about $1.4\omega_p$, because of the narrowing of the $|\text{Re}[\epsilon(\omega)]| < 1$ region at low frequencies. Eventually the $|\text{Re}[\epsilon(\omega)]| < 1$ region vanishes when strong magnetic fields is applied and there are totally attractive forces again. For the relatively small absorption of the magneto-optical material, the repulsive and restoring force may also exist, as demonstrated in Fig. 6, where the applied magnetic field is strong enough to result in wide $|\text{Re}[\epsilon(\omega)]| < 1$ region at low frequencies.

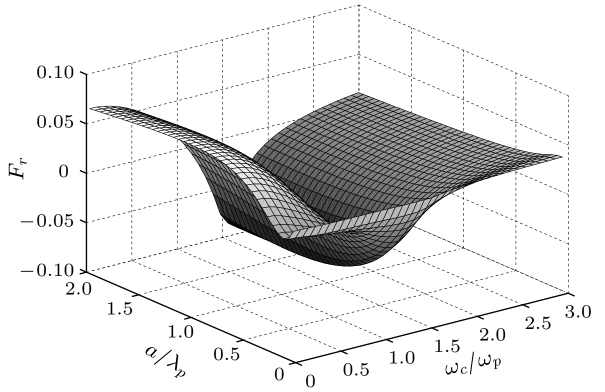

Fig. 6. The relative Casimir force F_r between an ordinary dielectric slab and a magneto-optical material slab as a function of the slab separation and the external magnetic field. For the magneto-optical material, the absorption is relatively small, $1/\tau = 0.1\omega_p$, and $\epsilon_0 = \epsilon_A = 2$. The slab thickness $d = \lambda_p$.

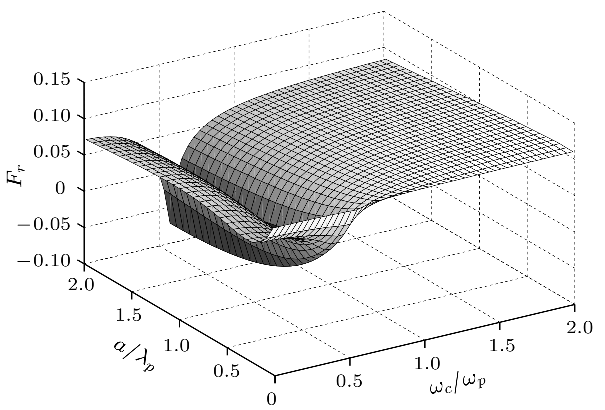

Fig. 7. The relative Casimir force F_r between an ordinary dielectric slab and a magneto-optical material slab (GaAs) as a function of the slab separation and the external magnetic field. For the magneto-optical material, $\epsilon_0 = 12.9$ (GaAs), $1/\tau = 0.8\omega_p$, $\epsilon_A = 2$ and the slab thickness $d = \lambda_p$.

Another method for obtaining the restoring force may be inspired: The external magnetic field needs to be adjusted to coincide with the varying slab separation. While the distance increases, the magnetic field should be accordingly lowered in order that the force is repulsive and attractive for the slab separation smaller and greater than certain distance, respectively. In this way one may construct the restoring force at almost any slab separation, provided that the repulsion can be found around that distance.

We can apply the above analysis to the real practical materials. In Fig. 7 the static dielectric constant of the semiconductor GaAs $\epsilon_0 = 12.9$ is taken,[18,19] and it is found that the repulsion and restoring force may clearly appear by adjusting the external magnetic field if suitable absorption is set. During the above discussion ω_p is adopted as a unit with which the frequencies are scaled, while the corresponding wavelength λ_p as a distance unit. When $\omega_p = 7.85 \times 10^{11}$ Hz is taken,[18,19] the corresponding wavelength is $\lambda_p = c/\omega_p = 3.8 \times 10^{-4}$ m. In the above figures, the repulsive force may exist at the minimum distance down to $a \sim 10^{-1}\lambda_p$. Thus, by the method of tuning the magnetic field while the slab separation varies, the restoring force can also obtained at possible closest distance $10^{-1}\lambda_p \sim 3.8 \times 10^{-5}$ m. Furthermore, the minimum distance for the repulsive and restoring force may even be shortened if other material is used. For instance, for InSb, $\omega_p = 1.6 \times 10^{13}$ Hz,[20] then the minimum distance $10^{-1}\lambda_p \sim 1.88 \times 10^{-6}$ m. Those separations are the typical distances between mechanical parts in micromachines and also are used in the Casimir force measurements.

In conclusion, we have studied the Casimir force direction tuned by the external magnetic field due to the magneto-optical Voigt effect. Between the ordinary dielectric slab and the magneto-optical material slab, repulsive Casmir force may be found. The frequency dependence of the modified permittivity is taken into account to analyze the forming of the repulsion. The restoring Casimir force, meaning the repulsion-attraction transition with the increasing distance, can also be obtained. Furthermore, we conclude that the repulsive and restoring force formed by adjusting the applied magnetic field may possibly exist at typical distances in MEMS and NEMS for real materials.

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