On refined volatility smile expansion in the Heston model

P. Friz, S. Gerhold, A. Gulisashvili, S. Sturm

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It is known that Heston's stochastic volatility model exhibits moment explosion, and that the critical moment s^{*} can be obtained by solving (numerically) a simple equation. This yields a leading order expansion for the implied volatility at large strikes: $\sigma_{BS}(k,T)^{2}T\sim \Psi (s^*-1) \times k$ (Roger Lee's moment formula). Motivated by recent "tail-wing" refinements of this moment formula, we first derive a novel tail expansion for the Heston density, sharpening previous work of Dr{\u{a}}gulescu and Yakovenko [Quant. Finance 2, 6 (2002), 443--453], and then show the validity of a refined expansion of the type $% \sigma_{BS}(k,T) ^{2}T=(\beta _{1}k^{1/2}+\beta_{2}+...)^{2}$, where all constants are explicitly known as functions of s^*, the Heston model parameters, spot vol and maturity T. In the case of the "zero-correlation" Heston model such an expansion was derived by Gulisashvili and Stein [Appl. Math. Opt., DOI: 10.1007/s002450099085]. Our methods and results may prove useful beyond the Heston model: the entire quantitative analysis is based on affine principles; at no point do we need knowledge of the (explicit, but cumbersome) closed form expression of the Fourier transform of $\log S_{T}$ (equivalently: Mellin transform of S_{T}%). Secondly, our analysis reveals a new parameter ("\textit{critical slope}"%), defined in a model free manner, which drives the second and higher order terms in tail- and implied volatility expansions.

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