

[Home](#) > [Journal](#) > [Earth & Environmental Sciences](#) > [IJG](#)[Indexing](#) [View Papers](#) [Aims & Scope](#) [Editorial Board](#) [Guideline](#) [Article Processing Charges](#)

IJG &gt; Vol.3 No.5A, October 2012

OPEN ACCESS

## Linearization of the Liouville Equation, Multiple Splits of the Chandler Frequency, Markowitz Wobbles, and Error Analysis

PDF (Size: 859KB) PP. 930-951 DOI : 10.4236/ijg.2012.325095

Author(s)

Cheh Pan

### ABSTRACT

The rotation of the physical Earth is far more complex than the rotation of a biaxial or slightly triaxial rigid body can represent. The linearization of the Liouville equation via the Munk and MacDonal perturbation scheme has oversimplified polar excitation physics. A more conventional linearization of the Liouville equation as the generalized equation of motion for free rotation of the physical Earth reveals: 1) The reference frame is most essential, which needs to be unique and physically located in the Earth; 2) Physical angular momentum perturbation arises from motion and mass redistribution to appear as relative angular momentum in a rotating Earth, which excites polar motion and length of day variations; 3) At polar excitation, the direction of the rotation axis in space does not change besides nutation and precession around the invariant angular momentum axis, while the principal axes shift responding only to mass redistribution; 4) Two inertia changes appear simultaneously at polar excitation; one is due to mass redistribution, and the other arises from the axial near-symmetry of the perturbed Earth; 5) The Earth at polar excitation becomes slightly triaxial and axially near-symmetrical even it was originally biaxial; 6) At polar excitation, the rotation of a non-rigid Earth becomes unstable; 7) The instantaneous figure axis or mean excitation axis around which the rotation axis physically wobbles is not a principal axis; 8) In addition to amplitude excitation, the Chandler wobble possesses also multiple frequency-splits and is slow damping; 9) Secular polar drift is after the products of inertia and always associated with the Chandler wobble; both belong to polar motion; 10) The Earth will reach its stable rotation only after its rotation axis, major principal axis, and instantaneous figure axis or mean excitation axis are all completely aligned with each other to arrive at the minimum energy configuration of the system; 11) The observation of the multiple splits of the Chandler frequency is further examined by means of exact-bandwidth filtering and spectral analysis, which confirms the theoretical prediction of the linearized Liouville equation. After the removal of the Gibbs phenomenon from the polar motion spectra, Markowitz wobbles are also observed; 12) Error analysis of the ILS data demonstrates that the incoherent noises from the Wars in 1920-1945 are separable from polar motion and removable, so the ILS data are still reliable and useful for the study of the continuation of polar motion.

### KEYWORDS

Liouville Equation; Polar Motion; Chandler Wobble; Markowitz Wobble; Error Analysis

### Cite this paper

C. Pan, "Linearization of the Liouville Equation, Multiple Splits of the Chandler Frequency, Markowitz Wobbles, and Error Analysis," *International Journal of Geosciences*, Vol. 3 No. 5A, 2012, pp. 930-951. doi: 10.4236/ijg.2012.325095.

### References

- [1] R. A. Becker, "Introduction to Theoretical Mechanics," McGraw-Hill, New York, 1954, 420 p.
- [2] W. T. Thomson, "Introduction to Space Dynamics," Wiley, New York, 1986, 317 p.
- [3] M. L. Smith, "Wobble and Nutation of the Earth," *Geophysical Journal Royal Astronomical Society*, Vol. 50, No. 1, 1977, pp. 103-140. doi:10.1111/j.1365-246X.1977.tb01326.x
- [4] M. L. Smith and F. A. Dahlen, "The Period and Q of the Chandler Wobble," *Geophysical Journal*

- [Open Special Issues](#)
- [Published Special Issues](#)
- [Special Issues Guideline](#)

[IJG Subscription](#)[Most popular papers in IJG](#)[About IJG News](#)[Frequently Asked Questions](#)[Recommend to Peers](#)[Recommend to Library](#)[Contact Us](#)

Downloads: 165,251

Visits: 393,693

[Sponsors, Associates, and Links >>](#)

- [5] S. R. Dickman, "The Rotation of the Ocean-Solid Earth System," *Journal of Geophysical Research*, Vol. 88, No. B8, 1983, pp. 6373-6394. doi:10.1029/JB088iB08p06373
- [6] W. H. Munk, "Polar Wandering: A Marathon of Errors," *Nature*, Vol. 177, No. 4508, 1956, pp. 551-554. doi:10.1038/177551a0
- [7] T. Gold, "Instability of the Earth's Axis of Rotation," *Nature*, Vol. 175, 1955, pp. 526-529. doi:10.1038/175526a0
- [8] W. H. Munk and G. J. F. MacDonald, "The Rotation of the Earth," Cambridge University Press, London, 1960, 323 p.
- [9] C. Pan, "Polar Wandering and the Earth's Dynamical Evolution Cycle," In: P. Melchior and S. Yumi, Eds., *Rotation of the Earth*, IAU Symposium No. 48, Reidel, 1972, pp. 206-211.
- [10] C. Pan, "Polar Motion of a Triaxial Earth and Dynamical Plate Tectonics," *Tectonophysics*, Vol. 25, 1975, pp. 1-40. doi:10.1016/0040-1951(75)90009-8
- [11] C. Pan, "The Earth's Rotation Instability, Plate Motion, and Geodynamics of the Mantle (Abstract)," *AGU Transactions*, Vol. 59, 1978, pp. 1202-1203.
- [12] C. Pan, "The Multiple-Frequency Chandler Wobble," *Journal of Physics of the Earth*, Vol. 30, No. 5, 1982, pp. 389-419. doi:10.4294/jpe1952.30.389
- [13] C. Pan, "Polar Instability, Plate Motion, and Geodynamics of the Mantle," *Journal of Physics of the Earth*, Vol. 33, No. 5, 1985, pp. 411-434. doi:10.4294/jpe1952.33.411
- [14] C. Pan, "Angular Momentum Perturbation, Polar Excitation, and Axial Near-Symmetry," *Geophysical Journal International*, Vol. 137, No. 1, 1999, pp. 139-148. doi:10.1046/j.1365-246x.1999.00782.x
- [15] C. Pan, "Observed Multiple Frequencies of the Chandler Wobble," *Journal of Geodynamics*, Vol. 44, No. 1-2, 2007, pp. 47-65.
- [16] R. S. Gross, "The Excitation of the Chandler Wobble," *Geophysical Research Letters*, Vol. 27, No. 15, 2000, pp. 2329-2332. doi:10.1029/2000GL011450
- [17] R. S. Gross, I. Fukumori and D. Menemenlis, "Atmospheric and Oceanic Excitation of the Earth's Wobbles during 1980-2000," *Journal of Geophysical Research*, Vol. 108, 2003, pp. 2370-2386. doi:10.1029/2002JB002143
- [18] R. S. Gross, "Earth Rotation: Long Period Variations," In: T. A. Herring, Ed., *Treatise of Geophysics*, Vol. 3, Elsevier, Oxford, 2007, pp. 239-294.
- [19] T. Sasao, S. Okubo and M. Saito, "A Simple Theory on the Dynamical Effects of a stratified fluid core upon Nutational Motion of the Earth," In: E. P. Fedorov, M. L. Smith and P. L. Bender, Eds., *Nutation and the Earth's Rotation*, IAU Symposium No. 78, Reidel, 1980, pp. 165-184.
- [20] T. Sasao and J. M. Wahr, "An Excitation Mechanism for the Free Core Nutation," *Geophysical Journal Royal Astronomical Society*, Vol. 64, No. 3, 1981, pp. 729-746. doi:10.1111/j.1365-246X.1981.tb02692.x
- [21] P. M. Mathews, T. A. Herring and B. A. Buffett, "Modeling of Nutation and Precession: New Nutation Series for Nonrigid Earth and Insights into the Earth's Interior," *Journal of Geophysical Research*, Vol. 107, No. B4, 2002.
- [22] C. Pan, "Non-Rigid Rotation, Secular Global Geodynamics and Free Nutation," *Physics of the Earth and Planetary Interiors*, 2012, in Press.
- [23] B. F. Chao, "On the Excitation of the Earth's Free Wobble and Reference Frames," *Geophysical Journal Royal Astronomical Society*, Vol. 79, No. 2, 1984, pp. 555-563. doi:10.1111/j.1365-246X.1984.tb02240.x
- [24] K. Lambeck, "The Earth's Variable Rotation: Geophysical Causes and Consequences," Cambridge University Press, London, 1980, 449 p. doi:10.1017/CBO9780511569579
- [25] H. Moritz and I. I. Mueller, "Earth Rotation: Theory and Observation," The Ungar Publishing Company, New York, 1988, 617 p.

- [26] P. M. Mathews, B. A. Buffett, T. A. Herring and I. I. Shapiro, " Forced Nutations of the Earth: Influence of Inner Core Dynamics, 1. Theory," *Journal of Geophysical Research*, Vol. 96, No. B5, 1991, pp. 8219-8242. doi:10.1029/90JB01955
- [27] G. M. T. D' Eleuterio and P. C. Hughes, " Dynamics of Gyro-Elastic Continua. AIAA/ASME/ASCE/AHS," 24th Structures, Structural Dynamics & Materials Conference, Lake Tahoe, 2-4 May 1983, 9 p.
- [28] R. T. H. Barnes, R. Hide, A. A. White and C. A. Wilson, " Atmospheric Angular Momentum Fluctuation, Length- of-Day Changes and Polar Motion," *Proceedings of the Royal Society*, Vol. 387, No. 1792, 1983, pp. 31-73.
- [29] M. J. Bell, R. Hide and G. Sakellarides, " Atmospheric Angular Momentum Forecasts as Novel Tests of Global Numerical Weather Prediction Models," *Philosophical Transactions of the Royal Society A*, Vol. 334, No. 1633, 1991, pp. 55-92. doi:10.1098/rsta.1991.0003
- [30] R. Hide and J. O. Dickey, " Earth' s Variable Rotation," *Science*, Vol. 253, No. 5020, 1991, pp. 629-637.
- [31] E. M. Gaposchkin, " Analysis of Pole Position from 1846 to 1970," In: P. Melchior and S. Yumi, Eds., *Rotation of the Earth*, IAU Symposium No. 48, Reidel, 1972, pp. 19-32.