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On the Net Cyclonic Circulation in Large Stratified Lakes*

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ABSTRACT

Schwab et al. have investigated a possible cause of long-term cyclonic circulation often observed in stratified lakes. They conclude that the adiabatic boundary condition applied to the bottom boundary leads to cyclonic circulation. In this note the authors emphasize that it is the combination of vertical mixing of density, the associated bottom and surface boundary conditions, and variable bottom topography that is important. The authors demonstrate the mechanism using a numerical model. Results also demonstrate the important role of the cross-shore flow (driven by vertical mixing of momentum) in setting the maximum azimuthal flow speed and the cross-shore structure of the azimuthal flow.

1. Introduction

Emery and Csanady (1973) have noted that many large stratified lakes and semienclosed seas and estuaries exhibit a mean cyclonic circulation during periods of stratification. Wunsch (1970) states that an adiabatic boundary condition for a stratified fluid over a sloping bottom implies that the fluid cannot remain in a state of rest since isotherms must intersect the bottom at right angles. Wunsch's idea is illustrated by Schwab et al. (1995), who model a parabolic lake of depth 100 m and width 100 km. There is no external forcing. Initially the lake is in a state of rest with a horizontally uniform thermocline between 10 and 20 m of depth. An adiabatic condition is imposed on all boundaries of the lake implying that the isotherms intersect the bottom at right angles. From the initial state of horizontally uniform stratification, the isotherms tilt to intersect the bottom normally. The downward sloping isotherms at the edge of the lake imply a surface-intensified cyclonic circulation in the Northern Hemisphere with a maximum of a few centimeters per second close to shore.

The purpose of this note is to emphasize that it is the combination of vertical mixing, the associated surface and bottom boundary conditions, and the variable depth that produces cyclonic circulation. We suggest that vertical mixing of cold water to the surface over deeper regions of the lake produces the doming thermocline that generates cyclonic flow. Without vertical mixing, there would be no doming of the thermocline. To understand the importance of variable depth in combination with adiabatic surface and bottom boundary conditions, consider the vertically averaged temperature field. Initially the lowest values are found where the water is deepest (see Fig. 1c for an example), indicating that if water were to be vertically mixed instantaneously, the resulting horizontal density gradients would produce a pressure gradient, which by the thermal wind equations drives a surface-intensified cyclonic circulation. The development of cyclonic circulation in the vertical average is the consequence of bottom friction, as we explain.

We repeat Schwab et al.'s study using a nonlinear eddy-resolving model based on Dietrich et al. (1987) and also include some additional experiments using a different bottom topography and one experiment with seasonal heating at the surface. The model is described in section 2 and the model results are presented in section 3. Section 4 provides a summary and discussion.

2. The model

We use a numerical model based on the SOMS approach (Dietrich et al. 1987; Dietrich and Ko 1994). We solve the nonlinear Navier–Stokes equations obtained after making the hydrostatic, Boussinesq, and rigid-lid approximations. The model uses a steplike topography and integrates the equations of motion on a fixed rectangular grid with 5-m resolution in the vertical and 2.5-km resolution in the horizontal. Constant eddy viscosity and diffusivity coefficients in the vertical and horizontal are used for each model run. The model solves for velocities and temperature. Salinity is set to a constant 0.2 ppt to represent fresh water. Vertical diffusion and viscosity coefficients were set to 3×10^{-5} and 2×10^{-5} m^2 s^{-1}, respectively, as in the uniform coefficient experiments of Schwab et al. (1995). Schwab et al. also use the Mellor and Yamada (1974) turbulent closure scheme, but obtain similar results using uniform coefficients. In our model runs, horizontal diffusivities and viscosities of 0, 5, and 50 m^2 s^{-1} have been used. Horizontal diffusivities and viscosities are set equal in all experiments.

The boundary conditions applied to the temperature are that the vertical temperature gradient is set to zero at the bottom and the horizontal temperature gradient normal to the boundary is set to zero at the sides of the lake (the latter only applies to model runs with nonzero horizontal diffusivity). Schwab et al. (1995) use the Princeton Ocean Model (POM: Blumberg and Mellor 1987). As they point out, the correct boundary condition at the lake bottom is that the isotherms intersect the bottom orthogonally. POM uses a sigma coordinate in the vertical. The no-normal flux condition is applied by setting the derivative of temperature with respect to sigma equal to zero at the bottom, the derivative being taken at fixed horizontal coordinates (Mellor 1992). This is equivalent to setting the vertical derivative of temperature equal to zero and is a valid approximation since the slope of the lake is weak, the lake width being 10^3 times greater than its depth.

For momentum, a quadratic bottom friction parameterization is employed with a drag coefficient equal to 0.002. In cases with nonzero horizontal viscosity, a no-slip boundary condition is applied at the horizontal walls; otherwise the boundary condition is one of no normal flow.

The model geometry consists of a circular lake with a depth of 100 m and a width of 100 km. The lake is centered at 45° N. Two model topographies with cylindrical symmetry are used. The first is a parabolic lake (Fig. 1a) and is the same model lake as used by Schwab et al. (1995). The second is a step shelf lake with two different depths (Fig. 1b), the shallow part being of depth 30 m, the deep part of depth 100 m. The lake is initially at rest, with a horizontally uniform temperature field. Initially, a linear temperature gradient between the depths of 10 and 20 m separates a 15°C surface layer from a 5°C deeper layer (Figs. 1a,b). The initial vertically averaged temperature field has horizontal gradients (Figs. 1c,d) in both cases, implying that vertical mixing in combination with adiabatic surface and bottom boundary conditions can be expected to generate flow in both cases. Unless otherwise stated, there is no applied external forcing implying no heat transfer with the environment. The only mechanism for initiating any circulation is vertical mixing in combination with the surface and bottom boundary conditions.

A test run with a flat-bottom topography was carried out. No circulation was found using the flat-bottom topography.

3. Results

a. Parabolic lake

We ran the model with the parabolic lake (Fig. 1a) to repeat the original model run by Schwab et al. (1995). The horizontal eddy diffusivity and viscosity are both set to zero. After 60 days, a cyclonic circulation is observed in the model. Surface water at the center of the lake is cooled by vertical diffusion more so than at the edges of the lake where the thickness of the initial 5°C cold water layer is thin. This results in a sinking of the isotherms at the edges of the lake (Fig. 2a). The corresponding vertically averaged circulation (Fig. 3) is cyclonic and reaches a maximum azimuthal velocity of 3 cm s^{-1} (Fig. 2c), comparable to Schwab et al. Figure 2b shows a vertical profile of azimuthal velocity. The circulation is cyclonic and surface intensified.

The surface intensification of the flow can be explained as follows. In the absence of bottom friction and as long as the flow maintains cylindrical symmetry, the vertically averaged flow remains zero. This can easily be demonstrated, assuming the nonlinear momentum advection terms are negligible (as is the case in most of our model experiments). Based on the thermal wind equations, one then expects to find cyclonic flow near the surface and anticyclonic flow at depth. Adding bottom friction to this flow generates bottom Ekman transport up the slope, away from the center of the lake, implying convergence of the vertically integrated transport toward the sides of the lake and divergence away from the center. As a consequence, the free surface rises at the sides of the lake, compared to the center, and generates vertically integrated cyclonic circulation that acts to reduce the anticyclonic flow at depth. Since our model makes the rigid-lid approximation, the barotropic adjustment associated with changing the free surface is filtered from our model equations and is assumed to occur instantaneously. In fact, when the rigid-lid approximation is made, convergence toward the sides of the lake is inconsistent with the vertically integrated continuity equation, implying that the generation of anticyclonic circulation at the bottom of the lake is strongly inhibited throughout the model integration. The net result is a surface-intensified cyclonic circulation, as can be seen in Fig. 2b.

Figure 2c is a transect of vertically averaged azimuthal velocity across the lake. Maximum velocities are centered roughly 10 km from the coast. This differs from Schwab et al. (1995, their Fig. 4) whose currents reach a maximum only a couple of kilometers from shore.

Model runs with nonzero horizontal diffusivity and viscosity coefficients were undertaken. Including horizontal mixing reduces overall current strength and spreads the current over a larger area. Maximum surface velocities after 60 days for model runs with horizontal diffusivity and viscosity coefficients of 0, 5, and 50 $m^2 s^{-1}$ are 11, 6, and 1.4 s^{-1} respectively. Increased vertical diffusivity increases circulation strength as noted by Schwab et al.

We carried out model runs using parabolic topography, but with half the original depth, twice the original depth, and twice the original width of the lake. While the topography differs, the initial temperature structure, vertical resolution, and horizontal resolution are the same in each case. The horizontal viscosity and diffusivity are set to zero. Two sets of experiments were carried out, one set using the full model, and a second set in which advection of the temperature field is withheld. Table 1 shows the maximum vertically averaged azimuthal velocity after 60 days in each case that was run. Schwab et al. suggest that the maximum vertically averaged azimuthal velocity depends inversely on the bottom slope. The 200-m and 100-m cases including advection of the temperature field are consistent with this dependence. The 50-m depth case is expected to have stronger circulation. However, in this case, the flow is baroclinically unstable, reducing current strength (we show an example of baroclinic flow instability when we consider the step shelf). For the run with a lake of double the width (100-m depth) and thus a weaker bottom slope, flow is weaker than the 100-m depth case contradicting the suggestion of Schwab et al. Reasons for this are discussed later.

With advection of the temperature field suppressed, model runs show the opposite dependence on bottom slope from that put forward by Schwab et al. (1995), with the strongest velocities being found when the bottom slope is greatest. It follows that advection of the temperature field has an important effect on the circulation. It should be noted that because of the radial symmetry (at least until flow instability sets in), advection is by the radial component of velocity. The latter can be regarded as a secondary Ekman circulation, generated by the vertical mixing of momentum associated with the azimuthal flow. The effect of the secondary circulation is to flatten isotherms, reducing the geostrophic shear. (Since geostrophic balance is a good approximation for the azimuthal flow, reduced vertical shear implies a reduced radial gradient of density because of thermal wind, and a corresponding flattening of the isopycnals.) In the 200-m depth case, the flow is confined in a narrow jet near shore when advection of the temperature field is suppressed. When advection of temperature is included, the radial velocity flattens out the radial density gradient, leading to a considerable reduction in the maximum velocity, as can be seen from Table 1. On the other hand, when the bottom slope is less, the flow field without advection is much more radially spread out, with the result that advection by the radial velocity is less effective at flattening isopycnals, leading to much less reduction in maximum velocity when advection is included, as can be seen from Table 1.

b. Step lake

In this section the model is run with the step lake topography (Fig. 1b). All side walls of the lake are vertical. Initially the isotherms are perpendicular to lake walls (Fig. 1b) and satisfy the side, bottom, and surface boundary conditions exactly. With time, vertical mixing causes the isotherms to dome in the center of the lake (Fig. 4a), leading to a cyclonic circulation at the shelf break (Fig. 5). In the absence of horizontal mixing, the circulation is baroclinically unstable (Figs. 4b,c, and 5). Horizontal mixing of 5 $m^2 s^{-1}$ is sufficient to suppress most of the flow instability in the step lake case.

If horizontal mixing is set at 50 m^2 s^{-1}, there is no flow instability, and maximum azimuthal currents in the step lake are slightly stronger than in the parabolic lake. In the case with no horizontal mixing, maximum azimuthal velocity in the step shelf case is less than half that in the parabolic case on account of the unstable flow found in the former case but not the latter.

4. Discussion

In a lake with a step topography (Fig. 1b) and for our choice of initial stratification, the adiabatic boundary conditions at the surface, walls, and bottom are satisfied exactly in the initial state. Vertical mixing is required to diffuse the thermocline, leading to a doming of the thermocline as the cold water in the deep part of the lake is mixed upward. The step-shelf topography case emphasizes the importance of vertical mixing for generating cyclonic circulation. As the thermocline diffuses, it eventually cannot satisfy the adiabatic boundary condition at the bottom without generating horizontal gradients and hence flow. This is despite the fact that in the initial state the adiabatic boundary conditions are satisfied exactly.

However, adiabatic boundary conditions are not necessarily needed to produce the doming of the thermocline. For the step shelf, if one were to uniformly heat the bottom of the lake and the bottom of the shelf, the shallow parts of the lake would heat up quicker than the center of the lake due to the shallow depth of the water column (assuming no heat input at the surface). A cyclonic circulation would be generated even though there is no longer a zero normal gradient boundary condition being applied at the bottom. While the imposition of the adiabatic condition at the bottom does play a role in realistic problems (Schwab et al. 1995), it is only part of the story. What is important is the combination of vertical mixing, the associated boundary conditions at the surface and at the bottom and the presence of variable depth.

Advection by the radial velocity can have a major impact on the maximum azimuthal velocity and is a key factor in determining the dependence of the maximum azimuthal velocity on the model parameters. Experiments show that azimuthal velocity is confined nearer to shore if advection of the temperature field is not included in the model. Advection of the temperature field is particularly important when the slope is steep and the horizontal mixing of momentum and temperature is zero. The offshore scale of the azimuthal flow is also determined by the location where the thermocline intersects bottom topography. The advective terms of the momentum equation are not important compared to the temperature advection terms, at least prior to the onset of flow instability. Basically, there is no change in circulation strength and pattern when advection of momentum is suppressed, except that the gyre pattern is slightly smoother.

The equation for maximum azimuthal velocity given by Schwab et al. (1995) appears to apply to a parabolic lake of fixed width where depth is varied, provided advection of the temperature field is included. On the other hand, if the width of the lake were increased by 1000 times, the slope of the lake would be almost flat. The equation in Schwab et al. would predict a very strong circulation since it relates current strength to the inverse of the bottom slope. However, when the width of the lake is increased, the horizontal gradient of the vertically averaged temperature field is reduced, and consequently cyclonic circulation is reduced. In the limit as the width of the lake increases to infinity, the parabolic lake becomes a flat-bottomed lake, there is no horizontal gradient in the vertical averaged temperature field and consequently no circulation.

We have considered in this study a mechanism for producing cyclonic circulation in the absence of any kind of external forcing. To relate our results to observed long-term circulations we show two plots of velocity contours in Lake Ontario from Simons and Schertzer (1989). Figure 6 shows the north–south cross section of time-averaged eastward current from May to August 1982 in Lake Ontario. Similarly Fig. 7 shows the time-averaged eastward current for the winter of 1983 (November 1982 to March 1983). Our model results of current cross section (Fig. 2b) compare well with the summer observations (Fig. 6). The model reproduces the vertical distribution of azimuthal current as well as the magnitude of surface circulation of around 6 cm s^{-1}. However, for winter, observed mean flow is basically barotropic (Fig. 7) and circulation is not just a simple cyclonic gyre. Thus, for the summer months, vertical diffusion of the thermocline is likely to be important for generating observed mean cyclonic flow.

A model run with uniform seasonal surface heating was run to investigate the effect of seasonal heating throughout the year. The lake was initially specified at a uniform temperature of 5°C to represent lake conditions at the end of winter. A sinusoidal-shaped heat flux was applied. During the warming phase, a cyclonic circulation is generated near shore (2 km) and progressively spreads farther offshore as heat is diffused to greater depths. However, during the cooling phase, a complex barotropic circulation sets in. The circulation of the lake during the cooling phase is beyond the scope of this comment and is not investigated further.

We have seen that the combination of vertical mixing, the surface and bottom boundary conditions, and variable bottom topography can together produce a cyclonic circulation in a stratified lake. Of these three factors we believe the combination of vertical mixing and variable bottom depth to be the key factor. Our results relate well to the long-term flow pattern in Lake Ontario for the summer months. Horizontal mixing reduces the strength of the overall cyclonic circulation.

Acknowledgments

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Tables

Table 1. Maximum vertical averaged azimuthal velocity (cm s^{-1}) after 60 days: 2 W refers to the case with twice the width; 200, 100, and 50 refer to the cases with depths 200, 100, and 50 m respectively;"adv. off" means that the advection of the temperature field is suppressed. Note that the 50-m depth case with advection exhibits a meandering flow at 60 days.

	200	100	50	2 W
Adv. on	2.39	4.12	3.69	3.91
Adv. off	6.26	5.47	2.74	3.18

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Figures

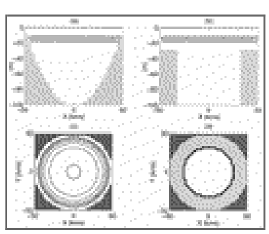

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Fig. 1. (a) Initial vertical temperature distribution and profile of the parabolic lake. (b) As in (a) but for the step lake. (c) Initial vertically averaged temperature field for the parabolic lake. Contour intervals are 1°C. The center of the lake is at 7.9°C. (d) As in (c) but for the step lake. The shallow region of the lake is at 12°C. The center of the lake is at 7.9°C.

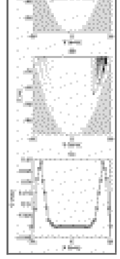

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Fig. 2. Vertical cross sections at $Y = 0$ (see Fig. 1c), of the parabolic lake model run at 60 days. (a) Temperature field: Contour intervals are 2°C and the deepest contour is 6°C. (b) Contour of velocity normal to the section with intervals 0.5 cm s^{-1}. Dotted lines represent out of the plane flow and solid lines represent flow into the plane. (c) Vertical average of the velocity in (b).

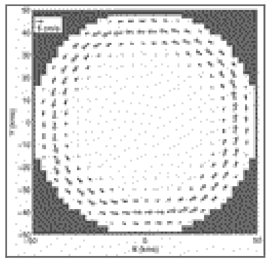

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Fig. 3. Vertically averaged circulation in parabolic lake model run at 60 days.

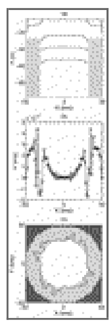

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Fig. 4. Cross sections ($Y = 0$) of the step lake model run at 60 days: (a) Temperature field; contour intervals are 2°C and the deepest contour is 6°C. (b) Vertically averaged azimuthal velocity. (c) Top view of vertically averaged temperature field. The one contour line is 11°C.

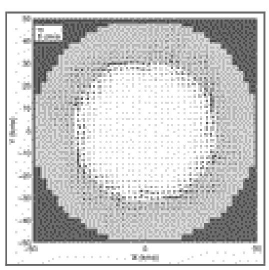

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Fig. 5. Step lake model run 60 days: Surface circulation.

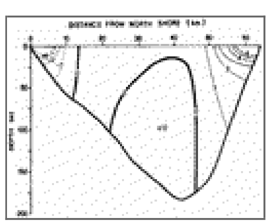

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Fig. 6. Time-averaged eastward circulation in Lake Ontario for summer of 1982. Reproduced from Simons and Schertzer (1989).

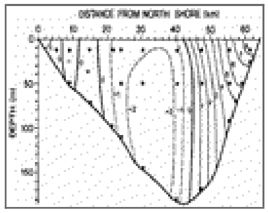

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Fig. 7. Time-averaged eastward circulation in Lake Ontario for winter 1982/83. Reproduced from Simons and Schertzer (1989).

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top