Journal of Convex Analysis, Vol. 6, No. 2, pp. 235-266 (1999)

Graph-Convex Mappings and K-Convex Functions

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Abstract: This paper studies global and local behavior of graph-convex set-valued mappings in finite-dimensional vector spaces. This is done in terms of recession mappings and graphical derivatives which are set-valued mappings whose graphs are convex cones. The main results are chain rules for computing the recession mapping and the graphical derivative of a composition of two set-valued mappings. The results on graph-convex mappings are applied to K-convex functions which are vector-valued generalizations of extended-real-valued proper convex functions. Many generalizations of classical results in convex analysis are obtained, along with a generalization of subdifferential calculus, in which the differential behavior of a function is described by a sublinear mapping that resembles the classical Jacobian. A particular advantage of this approach is that it leads to simple chain rules for compositions of vector-valued convex functions. The generality is reflected in the fact that most of the classical rules for computing recession functions and subdifferentials are obtained as special cases of the given chain rules. Some applications to mathematical programming and matrix analysis are given.

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