

论文

动力学平衡方程的Euler中点辛差分求解格式

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摘要 给出了动力学方程 $\{\pmb M\} \ddot{\{\pmb x\}} + \{\pmb C\} \dot{\{\pmb x\}} + \{\pmb K\} \{\pmb x\} = \{\pmb R\}$ 的二阶Euler中点隐式差分求解格式, 分保守系统、无阻尼受迫振动系统和阻尼系统3种情况, 讨论了算法中Jacobi矩阵 $\{\pmb A\}$ 的性质, 譬如 $\{\pmb A\}$ 是否为辛矩阵以及谱半径等. 对于无阻尼系统, 证明了无论是否存在外载荷, Jacobi 矩阵都是辛矩阵. 证明了辛矩阵的所有本征值的模为1, 其谱半径永远为1, 以及 $\delta = 0.5$ 和 $\alpha = 0.25$ 的Newmark算法就是Euler中点隐式差分格式, 对保守系统它们都是辛算法. 严格证明了Euler中点辛格式是严格保持系统能量的. 通过算例详细讨论了保辛算法用于求解非保守系统动态特性的优越性, 如广义保结构特性等; 分析了保辛算法的相位误差以及由其引起的系统的附加能量特性; 分析了保辛算法和 $\delta \neq 0.5$ 的Newmark算法的精度随着激励频率与系统固有频率比的变化情况等

关键词 [辛算法](#) [辛矩阵](#) [谱半径](#) [直接积分方法](#) [Jacobi矩阵](#)分类号 [0343](#)

Application of euler midpoint symplectic integration method for the solution of dynamic equilibrium equations

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Abstract

The dynamic equilibrium equations $\{\pmb M\} \ddot{\{\pmb x\}} + \{\pmb C\} \dot{\{\pmb x\}} + \{\pmb K\} \{\pmb x\} = \{\pmb R\}$ are solved by the Euler midpoint implicit integration method. The properties of Jacobi matrix of the algorithm are discussed in detail, and it is shown that Jacobi matrix independent of the external load vector $\{\pmb R\}$ is symplectic if $\{\pmb C\} = 0$, and the amplitude of all eigenvalues of symplectic matrix are equal to unity. It is proved that the Newmark method with $\delta = 0.5$ and $\alpha = 0.25$ is just the Euler midpoint implicit integration method; and for a conservative system, it is a structure-preserving algorithm, which means that the energy of the system is preserved through the solution process. Numerical analyses are carried out to illustrate the advantages of the symplectic algorithm in the solution of non-conservative systems. The accuracy of structure-preserving algorithm is not sensitive to the ratio of the frequency of the external force to that of the system, while the accuracy of Newmark algorithm with $\delta \neq 0.5$ is sensitive to that ratio.

Key words [symplectic algorithm](#) [symplectic matrix](#) [spectral radius](#) [direct integration method](#) [Jacobi matrix](#)

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