A scale-invariant probabilistic model based on Leibniz-like pyramids

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We introduce a family of probabilistic {\it scale-invariant} Leibniz-like pyramids and $(d+1)$-dimensional hyperpyramids ($d=1,2,3,...$), characterized by a parameter $\nu>0$, whose value determines the degree of correlation between N $(d+1)$-valued random variables. There are $(d+1)^N$ different events, and the limit $\nu\to\infty$ corresponds to independent random variables, in which case each event has a probability $1/(d+1)^N$ to occur. The sums of these N $\,(d+1)$-valued random variables correspond to a $d-$dimensional probabilistic model, and generalizes a recently proposed one-dimensional ($d=1$) model having $q-$Gaussians (with $q=(\nu-2)/(\nu-1)$ for $\nu \in [1,\infty)$) as $N\to\infty$ limit probability distributions for the sum of the N binary variables [A. Rodr\'{\i}guez {\em et al}, J. Stat. Mech. (2008) P09006; R. Hanel {\em et al}, Eur. Phys. J. B {\bf 72}, 263 (2009)]. In the $\nu\to\infty$ limit the $d-$dimensional multinomial distribution is recovered for the sums, which approach a $d-$dimensional Gaussian distribution for $N\to\infty$. For any ν, the conditional distributions of the $d-$dimensional model are shown to yield the corresponding joint distribution of the $(d-1)$-dimensional model with the same ν. For the $d=2$ case, we study the joint probability distribution, and identify two classes of marginal distributions, one of them being asymmetric and scale-invariant, while the other one is symmetric and only asymptotically scale-invariant. The present probabilistic model is proposed as a testing ground for a deeper understanding of the necessary and sufficient conditions for having q-Gaussian attractors in the $N\to\infty$ limit, the ultimate goal being a neat mathematical view of the causes clarifying the ubiquitous emergence of q-statistics verified in many natural, artificial and social systems.

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